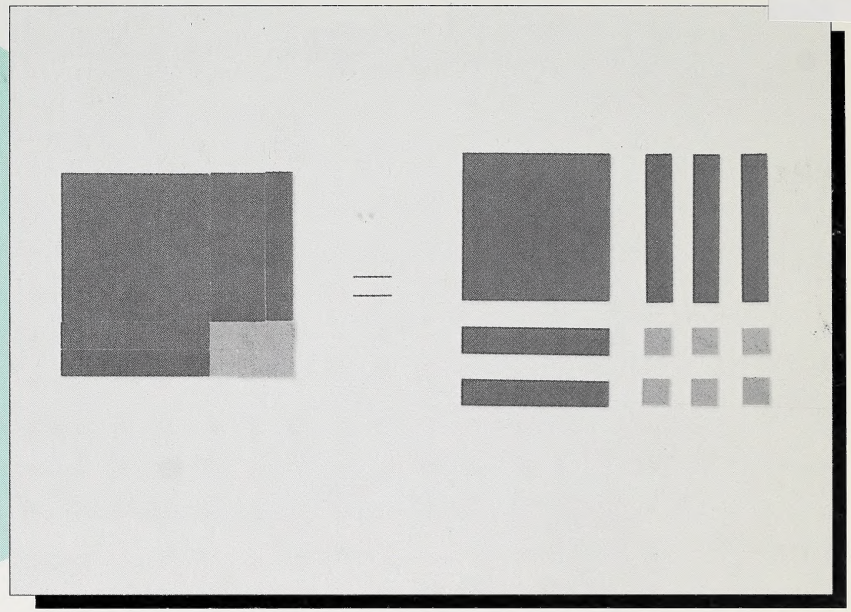
Mathematics 13



Distance Learning

Alberta EDUCATION

UNIT 2

Polynomials & Factoring

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Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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General Information

This information explains the basic layout of each booklet.

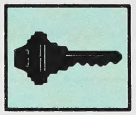
- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

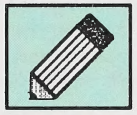
An explanation of what they mean is written beside each visual cue.

What You Already Know
- reviewing what you already know



Key Idea
- flagging important ideas

Audiotape
- learning by listening to an audiotape



Review
- studying previous concepts

Another View
- exploring different perspectives

Computer Software
- learning by using computer software

Introduction
- introducing the unit

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape

What Lies Ahead
- previewing the unit

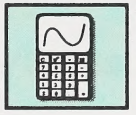
Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

Exploring the Topic
- actively learning new concepts

Extensions
- going on with the topic

Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator

What You Have Learned
- summarizing what you have learned

Mathematics 13

Course Overview

Mathematics 13 contains 6 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Polynomials and Factoring	**28%**
Unit 3 Geometry	18%
Unit 4 Coordinate Geometry and Graphing	18%
Unit 5 Relations	12%
Unit 6 Statistics	14%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Polynomials and Factoring

This unit covers topics dealing with polynomials and factoring. Each topic contains explanations, examples, and activities to assist you in understanding polynomials and factoring. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 2 Polynomials and Factoring

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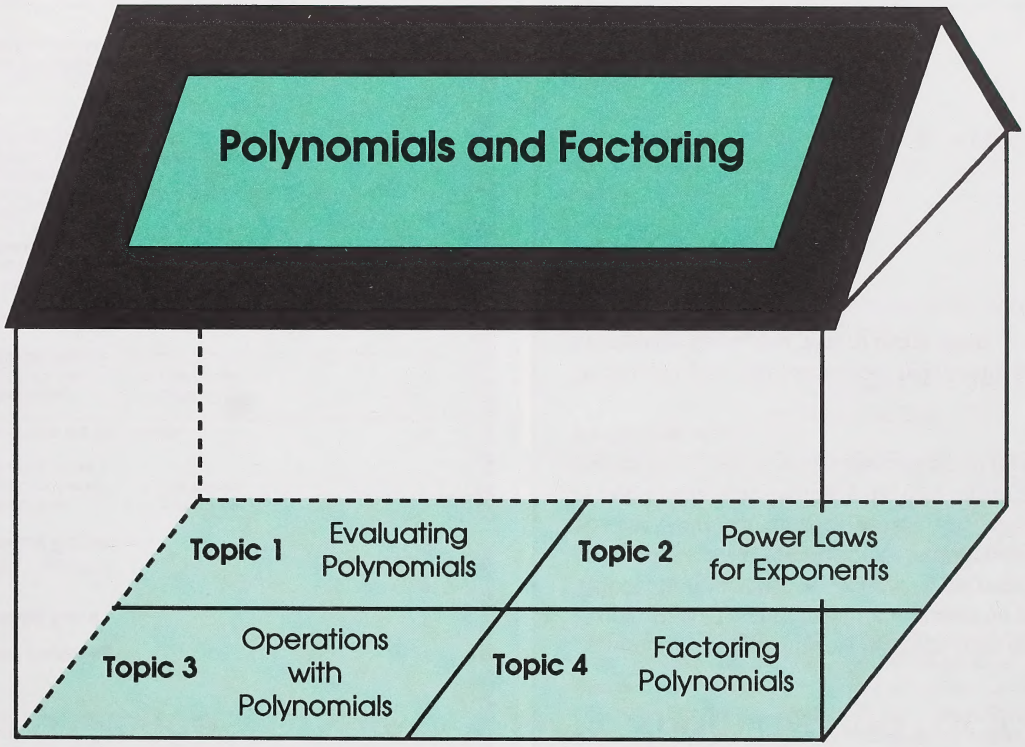
Polynomials and Factoring

Any construction ultimately depends upon its foundation. People do not want to build their houses on poor foundations. This is also true when working with polynomials and when factoring. A solid foundation or background is essential before you can build mathematical ideas. The foundation cornerstones of polynomials are basically the four topics in this unit. If you master these topics, your foundation will be solid and helpful for the rest of your life!

In this unit you will learn about polynomials, how to use and manipulate them, and how to factor them.


Polynomials and Factoring
Topic 1
Evaluating Polynomials
Topic 2
Power Laws for Exponents
Topic 3
Operations with Polynomials
Topic 4
Factoring Polynomials

What You Already Know

Recall the following.

Variables are symbols or letters which represent values or numbers. If you are given the values for the variables in an expression, you can evaluate the expression. This concept is often used to check your work in polynomials and to solve real problems.

Example 1

Evaluate $2x+y$ when $x=3$ and $y=2$.

Solution:

Substitute the given values of x and y. Remember that $2x$ means 2 times x.

$$\begin{aligned} 2x+y &= 2(3)+2 \\ &= 6+2 \\ &= 8 \end{aligned}$$

When you multiply powers with the same base, you add the exponents. This is called the Product Law.

Example 2

Simplify each of these expressions.

- $(2^3)(2^5)$

Solution:

$$\begin{aligned} (2^3)(2^5) &= 2^{3+5} \\ &= 2^8 \\ &= 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 \\ &= 256 \end{aligned}$$

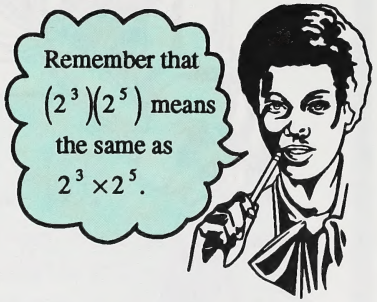
- $(-5)^2(-5)^3$

Solution:

$$\begin{aligned} (-5)^2(-5)^3 &= (-5)^{2+3} \\ &= (-5)^5 \\ &= (-5)(-5)(-5)(-5)(-5) \\ &= -3125 \end{aligned}$$

Do you remember these exponent laws?

- Product Law: $x^a \times x^b = x^{a+b}$
- Quotient Law: $\dfrac{x^a}{x^b} = x^{a-b}$



When you divide powers with the same base, you subtract the bottom exponent from the top exponent. This is called the Quotient Law.

Example 3

Simplify each of these expressions.

- $\frac{3^6}{3^2}$

  Solution:

$$\frac{3^6}{3^2} = 3^{6-2}$$
$$= 3^4$$
$$= 81$$

- $\frac{(-2)^8}{(-2)^4}$

  Solution:

$$\frac{(-2)^8}{(-2)^4} = (-2)^{8-4}$$
$$= (-2)^4$$
$$= (-2)(-2)(-2)(-2)$$
$$= 16$$

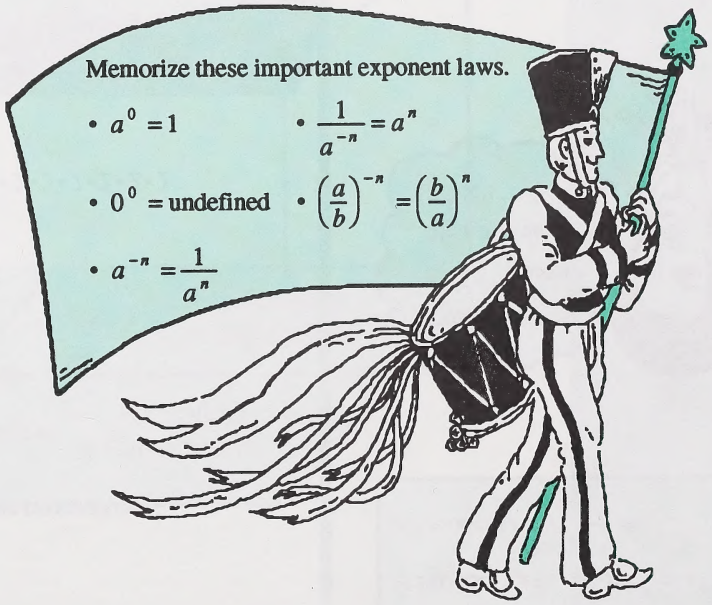
Any base, except zero, raised to the exponent zero equals 1.

Example 4

Evaluate $(-75)^0$.

Solution:

$$(-75)^0 = 1$$



Any base, except zero, raised to a negative exponent equals the reciprocal of the base with the exponent positive.

Example 5

Simplify each of the following.

- 2^{-1}

  Solution:

  $$2^{-1} = \frac{1}{2^1}$$
  $$= \frac{1}{2}$$

- 3^{-2}

  Solution:

  $$3^{-2} = \frac{1}{3^2}$$
  $$= \frac{1}{9}$$

- $\left(\frac{2}{3}\right)^{-2}$

  Solution:

  $$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2}$$
  $$= \frac{9}{4}$$

The greatest common factor of a group of numbers is the largest number that will divide evenly into every number of the group.

Example 6

Find the greatest common factor of the numbers 8, 12, 24, and 64.

Solution 1:

Write each of the numbers as a product of its prime factors.

$$8 = 2 \times 2 \times 2$$
$$12 = 2 \times 2 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

The common factors are 2×2. Therefore, the greatest common factor of the numbers 8, 12, 24, and 64 is $2 \times 2 = 4$.

Solution 2:

2	8	12	24	64
2	4	6	12	32
	2	3	6	16

Use the factors down the left side.

$$\text{G.C.F.} = 2 \times 2$$
$$= 4$$

Thus, the greatest common factor is 4.

Review

1. Evaluate each expression, given that $p=3$, $q=5$, and $r=-2$.

   a. $2p+3q$ b. $3r-3p$

   c. $2-2r$ d. $\frac{p+q}{r}$

   e. $3p-q$

2. Simplify each of the following completely.

   a. $(4^3)(2^2)$ b. $\left(\frac{1}{2}\right)^3(-2)^5$

   c. $(3^2)(5^0)$ d. 4^{-1}

   e. $\left(\frac{1}{3}\right)^{-2}$ f. 5^3+5^2

   g. 4^3+8^2 h. $(-2)^4(3^2)+4^2$

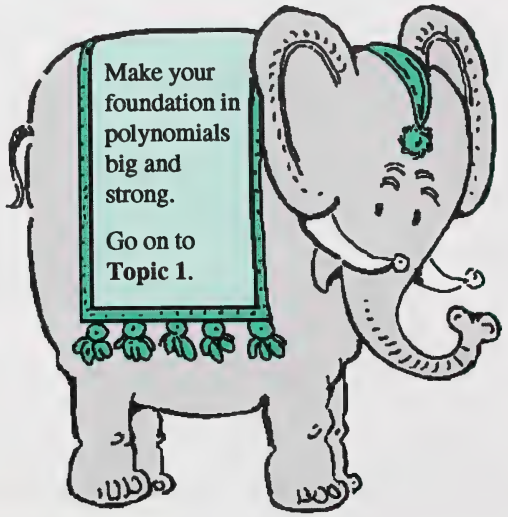
   i. $\left(\frac{3}{4}\right)^{-2}$

3. Find the greatest common factor for each of the following groups of numbers.

   a. 9, 18, 27 b. 18, 24, 30

Now go to the **Review** solutions in **Appendix A**.

If you had trouble with the **Review**, go to **Mathematics 9, Module 5**.



Topic 1 Evaluating Polynomials

Introduction

Building blocks can be made of a variety of materials such as wood, clay, stone, or brick. It is useful to know the composition of the blocks you use to build something.

Polynomial building blocks are composed of mathematical ideas. In this topic you are going to learn the meaning of the words used to describe these ideas. You are also going to learn to determine the value of these building blocks by replacing letters or variables with actual numbers.

What Lies Ahead

Throughout the topic you will learn to

1. identify the terminology of polynomials such as term, variable, monomial, and numerical coefficient

2. evaluate polynomials given the values of the variables

3. write polynomials which describe mathematical situations

Now that you know what to expect, turn the page to begin your study of evaluating polynomials.

Exploring Topic 1

Activity 1

Identify the terminology of polynomials such as term, variable, monomial, and numerical coefficient.

An algebraic **term** is an expression which can be connected to another by a plus or minus sign. The sign is a part of the term.

A few examples of terms are 3, -5, $2x$, x^4, and $-3x^2y$.

You do not call $3y-2x$ a term because it is actually made up of two terms which are $3y$ and $-2x$.

Variables are letters or symbols which represent a variety of values. You can substitute numerical values for the variables. This process is called substitution or evaluation.

Some terms are made up of a variable multiplied by a number. A special name is assigned to the number. The number is called the **numerical coefficient** or sometimes simply the coefficient.

Study these examples.

- In the term $2x$,
  2 is the numerical coefficient and
  x is the variable or the literal coefficient.
- In the term $-4x^2y$,
  -4 is the numerical coefficient while
  x and y are the variables or the literal coefficients.

Mathematicians have developed some special names for the polynomial building blocks.

A single term is called a **monomial.**

Some examples of monomials are $3x$, 5, and $-3xy^3$.

Two terms connected by a + sign or a – sign make a **binomial.**

Some examples of binomials are $2x+y$, $3x^2-5$, and $4y^2-3x$.

Three terms connected by + signs or – signs make a **trinomial.**

Two examples of trinomials are $2x^2-4x+3$ and $7x^3-4x^2+5xy^2$.

Groups of four terms and more are simply called polynomials of four or more terms.

A polynomial is a monomial or a sum of monomials.

All of the previous examples are polynomials. Polynomials can have any number of terms.

The term y really has a numerical coefficient even though it is not written. That numerical coefficient is 1. The term could be written as $1y$.

The term $-y$ also has a numerical coefficient. It is -1. So you could write $-y$ as $-1y$. In $3x-2y$, the numerical coefficient of y is -2, not 2.

The term y also has an exponent which is not written. The exponent is 1, so you could write y as y^1.

Now try these questions to see if you understand this material.

1. Identify which of the following are terms.

   a. $3x$ b. $2x-y$

   c. $-8x^3y^7z^2$ d. -3

   e. ax^2+b f. $\left(x^2y\right)^3$

2. For each part of question 1, list the variables.

3. For the polynomial $3x^2y-2xy^2+5y^3-7$, do the following:

   a. List the terms.

   b. Identify the numerical coefficient of each term.

4. Identify each of the following as a monomial, a binomial, or a trinomial.

   a. $2a^2b+4ab^2-c$ b. x^2+3

   c. $x+y-2$ d. -6

   e. y^2-16 f. ax^2+bx+c

For solutions to **Activity 1**, turn to **Appendix A, Topic 1.**

Activity 2

Evaluate polynomials given the values of the variables.

Variables can be replaced by numerical values. When you do this with a polynomial, you are evaluating the polynomial. The **order of operations** must be followed at all times.

Study the following example.

Example 1

Evaluate the following polynomials for the numerical values given.

- Evaluate x^2 when $x=3$.

  Solution:

  $$x^2 = (3)^2$$
  $$= 9$$

- Evaluate $-3y$ when $y=-4$.

  Solution:

  $$-3y = -3(-4)$$
  $$= 12$$

- Evaluate $2x-y$ when $x=3$ and $y=2$.

  Solution:

  $$2x-y = 2(3)-(2)$$
  $$= 6-2$$
  $$= 4$$

- Evaluate $3x^2-5xy-4$ when $x=2$ and $y=-3$.

  Solution:

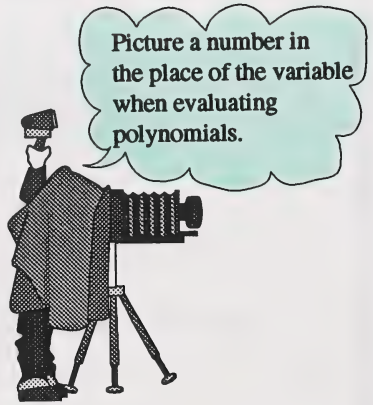
  $$3x^2-5xy-4 = 3(2)^2-5(2)(-3)-4$$
  $$= 3(4)-5(2)(-3)-4$$
  $$= 12+30-4$$
  $$= 38$$

Do at least four of the following questions.

Evaluate each expression, given that $x=4$ and $y=-2$.

1. $3x^2-4y$
2. $3y-x$
3. y^4-x^2
4. $2y^2-3xy+2$
5. $4y^2+4x$
6. $x^3y+x^2y^2+xy^3+y^4$

For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.



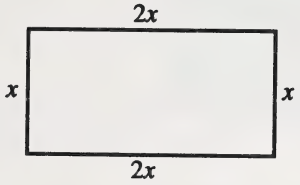
Activity 3

Write polynomials which describe mathematical situations.

You are often given situations which require expressions with variables to solve them. The following examples show how polynomials can be used to describe mathematical situations.

Example 2

A patio is shown in the diagram. What is the perimeter of the patio?



Solution:

The perimeter is the sum of all four sides.

$$P = 2x + x + 2x + x$$
$$= 6x$$

Example 3

What is the perimeter of an equilateral triangle with sides of length y?

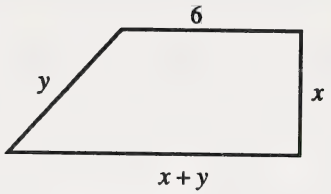
Solution:

An equilateral triangle has three sides of equal length so the perimeter is $y + y + y$ or $3y$.

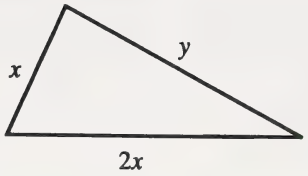
$$\therefore P = y + y + y$$
$$= 3y$$

Do a minimum of two of the following questions.

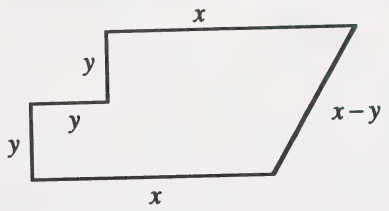
1. What is the expression for the perimeter of this figure?



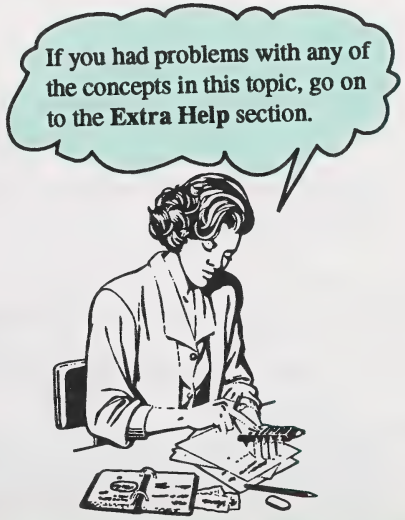
2. Write the expression for the perimeter of this triangle.



3. What is the perimeter of this figure?



For solutions to **Activity 3**, turn to **Appendix A**, **Topic 1**.



If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

A **polynomial** is one term or the sum of any number of terms.

Examine $6x^3 - 3x^2 + 4x - 1$.

This algebraic expression is called a polynomial and consists of four terms which are $6x^3$, $-3x^2$, $4x$, and -1.

Although every polynomial is considered to be a sum of terms, the addition sign is not always shown.

Thus, the previous expression may be written as
$6x^3 + (-3x^2) + 4x + (-1)$.

The terms of this sum are $6x^3$, $-3x^2$, $4x$, and -1.

Note that the sign is considered to be part of the term.

To simplify the naming of the terms of polynomials, addition signs are usually omitted.

For example, the terms of the polynomial $5y^2 + 9y + 8$ are $5y^2$, $9y$, and 8.

All like terms must be combined before the terms are counted to determine whether the polynomial is a monomial, a binomial, a trinomial, or some other type of polynomial.

- Note that $a^2 - 4ab$ is a two-term polynomial or a binomial.
- Also note that $x^3 - 3x^2 - 5x + 7$ is a four-term polynomial.
- However, what type of polynomial is $x^2 - 6x^2 + 3xy + 5xy - 4y^2 - 2y^2$?

The last polynomial contains like terms which can be combined to give a three-term polynomial. The terms x^2 and $-6x^2$ are like terms because the variable x^2 is the same in both terms. Therefore, they can be combined. The terms $3xy$ and $5xy$ are also like terms which can be combined. The same can be done to $-4y^2$ and $-2y^2$.

$$\therefore x^2 - 6x^2 + 3xy + 5xy - 4y^2 - 2y^2 = \left(x^2 - 6x^2\right) + (3xy + 5xy) + \left(-4y^2 - 2y^2\right)$$
$$= (1-6)x^2 + (3+5)xy + (-4-2)y^2$$
$$= -5x^2 + 8xy - 6y^2$$

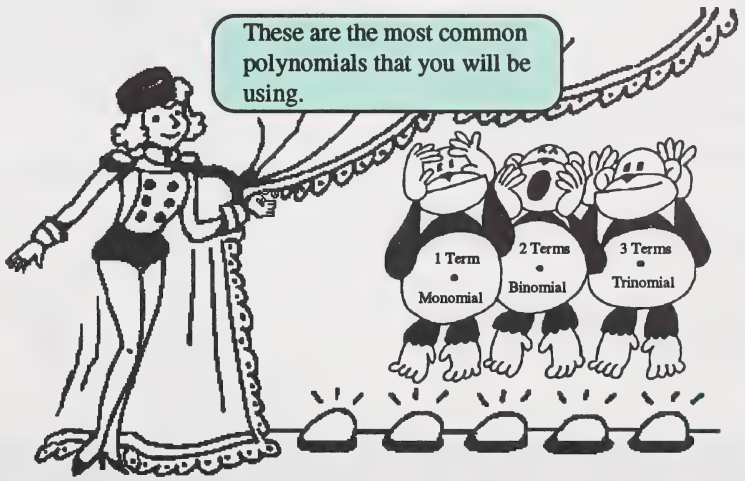
In simplified form this is a three-term polynomial or a trinomial.

Special names are given to polynomials with one, two, or three terms.

A polynomial with **one term** is called a **monomial**. Examples of monomials include $2x$, $4a^2b$, $3xy$, $\frac{1}{2}z^3$, and -7.

A polynomial with **two terms** is called a **binomial**. Examples of binomials include $5a+3b$, $-7x+9$, and $4m^2 - n$.

A polynomial with **three terms** is called a **trinomial**. Examples of trinomials include $3x+2y-5$, $-4a^2 + b - 6c^3$, and $2x^2 - 10xy + y^2$.



If you assign specific values to each of the variables in a given polynomial, you can find a numerical value for that polynomial.

Example 4

Evaluate the polynomial $4a+3b^2-5ac+2$ if $a=2$, $b=-3$, and $c=4$.

Solution:

In the given polynomial, replace a with 2, b with -3, and c with 4.

$$\begin{aligned} 4a+3b^2-5ac+2 &= 4(2)+3(-3)^2-5(2)(4)+2 \\ &= 8+3(9)-5(8)+2 \\ &= 8+27-40+2 \\ &= (8+27+2)-40 \\ &= 37-40 \\ &= -3 \end{aligned}$$

- Monomials have one term.
- Binomials have two terms.
- Trinomials have three terms.
- The rest are named as four-term polynomials, five-term polynomials, six-term polynomials, and so on.

Now try the following questions.

1. List the terms of each polynomial. Use commas to separate the terms.

   a. $4x^2-5xy+7y^2$

   b. $a-b^2+3ab-5a^2b^2+7$

   c. $z^4-3z^2-9z+12$

2. Classify each of the following polynomials as a monomial, a binomial, a trinomial, or a four-term polynomial.

   a. $3a^2-5a+6$

   b. $\dfrac{-a^2}{4}$

   c. x^3-3x^2+2x-4

   d. $m-2n$

   e. $4abc$

   f. $a^2-2ab+b^2$

   g. b^5-3a^2

   h. $x^2y^2-3x^2+y^2-4xy$

   i. $6r^2s^3t^4$

   j. $4m^2-8mn+2n^2$

3. If $x=3$, $y=-1$, and $z=2$, evaluate each of the following expressions.

   a. $4x-2y+z$

   b. x^2-y^4+z

   c. x^3-z^3

   d. $2xy+3xz$

For solutions to **Extra Help**, turn to **Appendix A, Topic 1.**

Extensions

An extension of evaluating polynomials is the use of formulas. When you are finding the area of a square, a rectangle, or a trapezoid, you are really evaluating a polynomial. The formulas for finding the areas of the figures mentioned are given.

- The formula for the area of a square is $A=s^2$.
- The formula for the area of a rectangle is $A=lw$.
- The formula for the area of a trapezoid is $A=\frac{1}{2}a(b_1+b_2)$.

Another formula which is an example of evaluating polynomials is the compound interest formula. This formula reads as $A=P(1+r)^n$,

where A = amount of interest and principal,
P = principal or money invested,
r = rate per compounding period, and
n = number of times the interest is taken into account.

Try the following question.

Calculate the amount of interest received after four years if $500.00 is invested at $15\frac{1}{4}\%$ per annum compounded yearly.

To find A, use the formula $A=P(1+r)^n$, where $P=500$, $r=15\frac{1}{4}\%$ or 0.1525, and $n=4$.

For solutions to **Extensions**, turn to **Appendix A, Topic 1.**

Topic 2 Power Laws for Exponents

Introduction

When a carpenter is going to use a tool or some building materials, it is a good idea to become familiar with that tool or those materials. In order to become good at your job, you need to practise skills already learned, learn new skills, and apply yourself to tasks using these old and new skills.

In this topic you will learn some fundamental concepts. In order to become good at the mathematics in this unit, it is very important that you work hard at mastering the skills in this topic.

What Lies Ahead

Throughout the topic you will learn to

1. use the Product Law of exponents
2. use the Power-of-a-Power Law of exponents
3. use the Quotient Law of exponents
4. use zero exponents and negative exponents

Now that you know what to expect, turn the page to begin your study of power laws for exponents.

Exploring Topic 2

Activity 1

Use the Product Law of exponents.

You will be using some mathematical names throughout this topic and therefore it is important to recognize these names. Here are some commonly used names to know and remember.

The expression x^m is a **power**.

In x^m the **exponent** is m.

In x^m the **base** is x.

Now look at some simple powers and see how the Product Law of exponents is developed.

The power 2^3 means $2 \times 2 \times 2$.

The power 2^2 means 2×2.

If you have a question like $2^3 \times 2^2$, you can write out the question in expanded form as shown.

$$2^3 \times 2^2 = \underbrace{2 \times 2 \times 2}_{2^3} \times \underbrace{2 \times 2}_{2^2}$$

You can write the expanded form more simply as a power since the number 2 multiplied by itself five times can be written as 2^5.

Therefore, $2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2$

$$= 2^5.$$

Compare the exponents in the question and the answer to see if you can find a relationship.

$$2^3 \times 2^2 = 2^5$$

It appears that the exponents in the question are added to give the exponent in the answer.

Try another example like $(3^4)(3^5)$. Write it in expanded form.

$$(3^4)(3^5) = \underbrace{(3 \times 3 \times 3 \times 3)}_{3^4}\underbrace{(3 \times 3 \times 3 \times 3 \times 3)}_{3^5}$$

$$= 3^9$$

Again it seems true that you add the exponents 4 and 5 to get 9.

Try one with a literal base. A **literal base** is a base that is a variable, not a number. In $(x^2)(x^3)$ the bases are literal.

Write $(x^2)(x^3)$ in expanded form.

$$(x^2)(x^3) = \underbrace{(x \times x)}_{x^2}\underbrace{(x \times x \times x)}_{x^3}$$

$$= x^5$$

This addition of exponents seems to work with literal bases as well. Do you think you can find an example that will prove this addition of exponents wrong? Try it!

The **Product Law of powers** with the same base is as follows:

$$x^a \cdot x^b = x^{a+b}$$

If the **bases are the same**, a product of powers can be written as that base raised to the **sum** of the exponents.

Of the following questions, do question 1 and any three of the others.

Simplify each of the following and leave your answers in exponent form.

1. Verify that $(x^a)(x^b) = x^{a+b}$ using the specific example $3^2 \times 3^3$.

2. $(4^2)(4^3)$

3. $(3^5)(3^9)$

4. $(x^5)(x^6)$

5. $(x^2)(y^2)$

6. $(2^2)(2^3)(2^4)$

7. $(x^2)(x)$

For solutions to **Activity 1**, turn to **Appendix A**, **Topic 2**.

Activity 2

Use the Power-of-a-Power Law of exponents.

Look at a simple power of a power and see how the Power-of-a-Power Law for exponents is developed.

$$\begin{aligned}(2^2)^3 &= (2^2)(2^2)(2^2) \\ &= \underbrace{2\times 2}_{2^2} \times \underbrace{2\times 2}_{2^2} \times \underbrace{2\times 2}_{2^2} \\ &= 2^6\end{aligned}$$

What appears to be the relationship between the exponents in the question and the exponent in the answer?

It appears that the exponents 2 and 3 are multiplied to get 6.

Try another example such as $(3^2)^5$.

$$\begin{aligned}(3^2)^5 &= (3^2)(3^2)(3^2)(3^2)(3^2) \\ &= \underbrace{3\times 3}_{3^2} \times \underbrace{3\times 3}_{3^2} \times \underbrace{3\times 3}_{3^2} \times \underbrace{3\times 3}_{3^2} \times \underbrace{3\times 3}_{3^2} \\ &= 3^{10}\end{aligned}$$

This follows the same pattern since $2 \times 5 = 10$.

Try an example with a literal base such as $\left(y^3\right)^2$.

$$\begin{aligned}\left(y^3\right)^2 &= \left(y^3\right)\left(y^3\right)\\ &= \underbrace{y\times y\times y}_{y^3}\times\underbrace{y\times y\times y}_{y^3}\\ &= y^6\end{aligned}$$

Since $3\times 2 = 6$, this works in the same way.

The general expression for the **Power-of-a-Power Law** can be written as follows:

$$\left(x^a\right)^b = x^{ab}$$

Do question 1 and any of the other three questions.

Simplify the following and leave your answers in exponent form.

1. Verify the law $\left(x^a\right)^b = x^{ab}$ using a specific example. Select the base and the exponents yourself.

2. $\left(3^3\right)^4$

3. $\left(5^2\right)^6$

4. $\left(x^4\right)^5$

5. $\left(x^2\right)^4$

6. $\left(y^4\right)^6$

7. $\left(10^2\right)^0$

For solutions to **Activity 2**, turn to **Appendix A, Topic 2.**

Activity 3

Use the Quotient Law of exponents.

Now look at a simple division of powers and see how the Quotient Law for exponents is developed.

$$\frac{2^5}{2^2} = \frac{2\times2\times2\times2\times2}{2\times2}$$

$$= 2^3$$

Note that the common factors were divided.

$$\frac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times2\times2\times2}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}} = 2\times2\times2$$

$$= 2^3$$

Again, look at the relationship between the exponents.

$$5 - 2 = 3$$

It appears that you subtract the bottom exponent from the top exponent when you divide powers with the same base.

Look at an example using literal bases.

$$\frac{x^6}{x^4} = \frac{\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(x)}}(x)(x)}{\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}}$$

$$= x^2$$

In general the **Quotient Law of powers** is written as follows:

$$\frac{x^a}{x^b} = x^{a-b}, \text{ where } x \neq 0$$

Why do you suppose that x cannot equal zero?

Try dividing anything by zero on your calculator! You should get an E on the display. This E stands for error.

Of the following questions, do question 1 and any other three.

Simplify each of the following.

1. Verify $x^a \div x^b = x^{a-b}$. Use your own example.

2. $2^6 \div 2^2$

3. $7^9 \div 7^5$

4. $x^8 \div x^4$

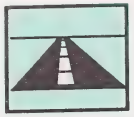
5. $\dfrac{y^7}{y^3}$

6. $x^3 \div x$

7. $\dfrac{x^6 y^4}{x^3 y^3}$

For solutions to **Activity 3**, turn to **Appendix A, Topic 2.**

Activity 4



Use zero exponents and negative exponents.

Look carefully at the following calculation of $2^3 \div 2^3$.

$$\frac{2^3}{2^3} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}}$$

$$= \frac{1 \times 1 \times 1}{1 \times 1 \times 1}$$

$$= \frac{1}{1}$$

$$= 1$$

Using the Quotient Law, you get the following:

$$\frac{2^3}{2^3} = 2^{3-3}$$

$$= 2^0$$

Since $\frac{2^3}{2^3} = 1$ and $\frac{2^3}{2^3} = 2^0$, then $2^0 = 1$.

Now try one with a literal base.

$$\frac{x^4}{x^4} = \frac{\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(x)}}}{\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}}$$

$$= \frac{1 \times 1 \times 1 \times 1}{1 \times 1 \times 1 \times 1}$$

$$= \frac{1}{1}$$

$$= 1$$

$$\frac{x^4}{x^4} = x^{4-4}$$

$$= x^0$$

Since $\frac{x^4}{x^4} = 1$ and $\frac{x^4}{x^4} = x^0$, here again $x^0 = 1$.

In general the **Zero Law for exponents** is as follows:

$x^0 = 1$, where $x \neq 0$

What happens if you divide $2^2 \div 2^5$?

$$\frac{2^2}{2^5} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times 2 \times 2 \times 2}$$

$$= \frac{1}{2 \times 2 \times 2}$$

$$= \frac{1}{2^3}$$

The Quotient Law allows you to subtract exponents, so you get the following:

$$\frac{2^2}{2^5} = 2^{2-5}$$
$$= 2^{-3}$$

Since $\frac{2^2}{2^5} = \frac{1}{2^3}$ and $\frac{2^2}{2^5} = 2^{-3}$, then $\frac{1}{2^3} = 2^{-3}$ or $2^{-3} = \frac{1}{2^3}$.

What happens when literal bases are used?

$$x^3 + x^7 = \frac{\overset{1}{(x)}\overset{1}{(x)}\overset{1}{(x)}}{\underset{1}{(x)}\underset{1}{(x)}\underset{1}{(x)}(x)(x)(x)(x)}$$
$$= \frac{(1)(1)(1)}{(1)(1)(1)(x)(x)(x)(x)}$$
$$= \frac{1}{x^4}$$

The result according to the Quotient Law is as follows:

$$\frac{x^3}{x^7} = x^{3-7}$$
$$= x^{-4}$$

Therefore, $x^{-4} = \frac{1}{x^4}$.

In general the rule for negative exponents is as follows:

$$x^{-a} = \frac{1}{x^a}, \text{ where } x \neq 0$$

Look at the following expression.

$$1 + \frac{2^2}{2^5}$$

Since you are dividing by a fraction, you invert and multiply.

$$1 + \frac{2^2}{2^5} = 1 \times \frac{2^5}{2^2}$$

Now apply the Quotient Law of powers.

$$1 \times \frac{2^5}{2^2} = 1 \times 2^{5-2}$$
$$= 1 \times 2^3$$
$$= 2^3$$

Therefore, $1 + \frac{2^2}{2^5} = 2^3$.

Another way to write $1 + \frac{2^2}{2^5}$ is as follows:

$$1 + 2^{2-5} = 1 + 2^{-3}$$
$$= \frac{1}{2^{-3}}$$

$1+\frac{2^2}{2^5}=2^3$ and $1+\frac{2^2}{2^5}=\frac{1}{2^{-3}}$

Therefore, $2^3=\frac{1}{2^{-3}}$.

In general this rule can be expressed as follows:

$\frac{1}{x^{-a}}=x^a$, where $x\neq 0$

Look at the following examples.

Example 1

Write 3^{-6} using a positive exponent.

Solution:

Using the rule, you get $3^{-6}=\frac{1}{3^6}$.

Example 2

Simplify 375^0.

Solution:

Using the Zero Law for exponents, $375^0=1$.

Now apply these laws and rules in the questions that follow.

Do questions 1 to 3 and any other three questions.

Simplify each of the following completely and write your answers with **only positive** exponents.

1. Verify $x^0=1$ when $x\neq 0$ using a specific example.

2. Verify $x^a=\frac{1}{x^{-a}}$ using a specific example.

3. Vertify $x^{-a}=\frac{1}{x^a}$ using a specific example.

4. $\frac{3^{-5}}{4^0}$

5. $\frac{y^5}{x^{-3}z^0}$

6. $\frac{\left(2x^9y^7\right)^0}{x^{-2}}$

7. $\frac{25^0}{4^{-2}}$

8. $\frac{x^{-3}}{x^{-5}}$

9. $\frac{2^{-4}}{3^{-2}}$

For solutions to **Activity 4**, turn to **Appendix A, Topic 2.**

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Remember the basic power properties:

- When you multiply, add the exponents.

exponents

$x^a \times x^b = x^{a+b}$ ← Add the exponents.

same base

For example, $x^3 \times x^4 = x^{3+4}$

$= x^7$

- When you divide, subtract the exponents.

exponents

same base $\frac{x^a}{x^b} = x^{a-b}$ ← Subtract the exponents.

For example, $\frac{x^5}{x^3} = x^{5-3}$

$= x^2$

- To find the power of a power, multiply the exponents.

Multiply the exponents.

↓

$(x^a)^b = x^{ab}$

For example, $(x^3)^5 = x^{3\times5}$

$= x^{15}$

- Any base with a zero exponent is equal to 1. However, the base cannot be zero since 0^0 is undefined.

For example, $5^0 = 1$

$x^0 = 1$

$3^0 = 1$

$a^0 = 1$

- When a power is moved from the numerator to the denominator or from the denominator to the numerator, the sign of its exponent changes.

  For example,

$$x^{-2} = \frac{x^{-2}}{1} = \frac{1}{x^2} \qquad \frac{1}{x^{-2}} = \frac{x^2}{1} = x^2$$

Study the following examples.

Example 3

Simplify $3^2 \times 3^5$.

Solution:

$$3^2 \times 3^5 = 3^{2+5} = 3^7$$

Example 4

Simplify $2^5 \times 3^5$.

Solution:

This expression cannot be simplified because the **bases are not the same.**

Example 5

Simplify $a^2 \times b^3 \times a^4$.

Solution:

$$a^2 \times b^3 \times a^4 = a^{2+4} b^3 = a^6 b^3$$

Example 6

Simplify $\frac{5^3}{5^2}$.

Solution:

$$\frac{5^3}{5^2} = 5^{3-2} = 5^1 = 5$$

Example 7

Simplify $\frac{3^5}{2^5}$.

Solution:

This expression cannot be simplified because the **bases are not the same.**

Example 8

Simplify $\frac{a^3}{a^3}$.

Solution:

$$\frac{a^3}{a^3} = a^{3-3}$$
$$= a^0$$
$$= 1$$

Example 9

Simplify $\frac{a^{-2}b^3}{ac^{-3}}$.

Solution:

$$\frac{a^{-2}b^3}{ac^{-3}} = \frac{b^3 \times c^3}{a \times a^2}$$
$$= \frac{b^3 c^3}{a^3}$$

Now try these questions.

1. Simplify each of the following.

   a. $6^3 \times 6^2$ b. $5^3 \times 5^{-2}$ c. $x^3 \times x^5$

2. Simplify each of the following.

   a. $\frac{5^4}{5^3}$ b. $\frac{x^6}{x^2}$

   c. $\frac{x^3}{x^{-2}}$ d. $\frac{x^0}{x^{-1}}$

3. Simplify each of the following.

   a. $\left(3^2\right)^5$ b. $\left(x^2\right)^7$ c. $\left(x^{-2}\right)^7$

For solutions to **Extra Help**, turn to **Appendix A, Topic 2.**

Extensions

In simplifying some powers with variable bases, you may have to use several power properties to complete the simplification process.

Example 10

Simplify $\left(\frac{x^5}{x^2}\right)^2$.

Solution:

$$\left(\frac{x^5}{x^2}\right)^2 = \left(x^{5-2}\right)^2$$ (Use the Quotient Law of exponents.)

$$= \left(x^3\right)^2$$

$$= x^{3\times 2}$$ (Use the Power-of-a-Power Law of exponents.)

$$= x^6$$

Example 11

Simplify $\frac{\left(m^3\right)^{-2} \times \left(m^2\right)^5}{\left(m^{-3}\right)^{-2}}$.

Solution:

$$\frac{\left(m^3\right)^{-2} \times \left(m^2\right)^5}{\left(m^{-3}\right)^{-2}} = \frac{m^{3(-2)} \times m^{2\times 5}}{m^{(-3)(-2)}}$$ (Use the Power-of-a-Power Law of exponents.)

$$= \frac{m^{-6} m^{10}}{m^6}$$

$$= \frac{m^{-6+10}}{m^6}$$ (Use the Product Law of exponents.)

$$= \frac{m^4}{m^6}$$

$$= m^{4-6}$$ (Use the Quotient Law of exponents.)

$$= m^{-2}$$

$$= \frac{1}{m^2}$$

Now try the following.

1. Simplify $\left(x^3 \times x^5\right)^3$.

2. Simplify $\left(\frac{x^3}{x^5}\right)^4$.

3. Simplify $\frac{\left(a^3\right) \times \left(a^2\right)^{-2}}{a^2}$.

4. Simplify $\frac{\left(a^3\right)^0 \times a^2}{\left(a^{-1}\right)^2}$.

5. Simplify $\left(\frac{y^3}{y^{-2}}\right)^4$.

For solutions to **Extensions**, turn to **Appendix A, Topic 2.**

Remember that

- $x^1 = x$
- $a^0 = 1$ except when $a = 0$

Topic 3 Operations with Polynomials

Introduction

Calculators are very good at performing operations involving numbers. However, the average calculator is not very good at working with variables. It takes a sophisticated calculator and a knowledgeable user to perform calculations involving polynomials.

In this unit you will study some of the basic operations involving polynomials. Maybe someday you will have a calculator that can help you with these operations.

What Lies Ahead

Throughout the topic you will learn to

1. add and subtract polynomials and arrange terms in alphabetical and descending order with respect to degree
2. multiply polynomials by monomials
3. multiply binomials by binomials
4. divide polynomials by monomials

Now that you know what to expect, turn the page to begin your study of operations with polynomials.

Exploring Topic 3

Activity 1

Add and subtract polynomials and arrange terms in alphabetical and descending order with respect to degree.

A more concrete or direct view of the addition and subtraction of polynomials can be achieved using polynomial building blocks or tiles. These tiles will be used in multiplication, division, and factoring so this is an excellent place to start learning what they are and how they are used.

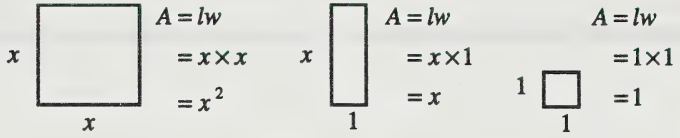
You are going to use three different tiles to represent polynomial terms. Study these tiles very carefully.

flat strip unit

Each of these will represent a different term. The lengths of the sides are given and the areas are calculated for you.

Use the following formula:

Area = length × width or $A = lw$ in short form



Check for yourself to see if the areas given are correct.

Turn to **Appendix B** at the end of this unit and find the pages with these polynomial building blocks or tiles. The set includes six flats (x^2), twenty strips (x), and fifty units (1). Cut out all of these tiles. It may be handy to store them in a large envelope so they do not get lost.

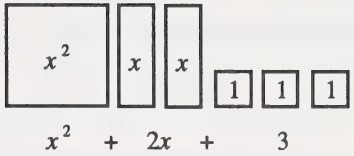
All done? Great! Now you can proceed.

Build the polynomial $x^2 + 2x + 3$ using your polynomial building blocks or tiles.

You need the following:

- one flat or one x^2
- two strips or two x's
- three units or three 1's

You should have the tiles arranged as shown.



Example 1

What polynomial does each group represent?

Solution:

The tiles represent the polynomial $2x^2+4x+1$.

Solution:

The tiles represent the polynomial $4x^2+2x+4$.

Do the following questions.

1. Use your polynomial tiles to build each of the following.

   a. $3x^2+5x+7$ b. x^2+x+2

2. What polynomial does each of the following arrangements represent?

   a.

   b.

For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

Now you will add polynomials using these same tiles.

Example 2

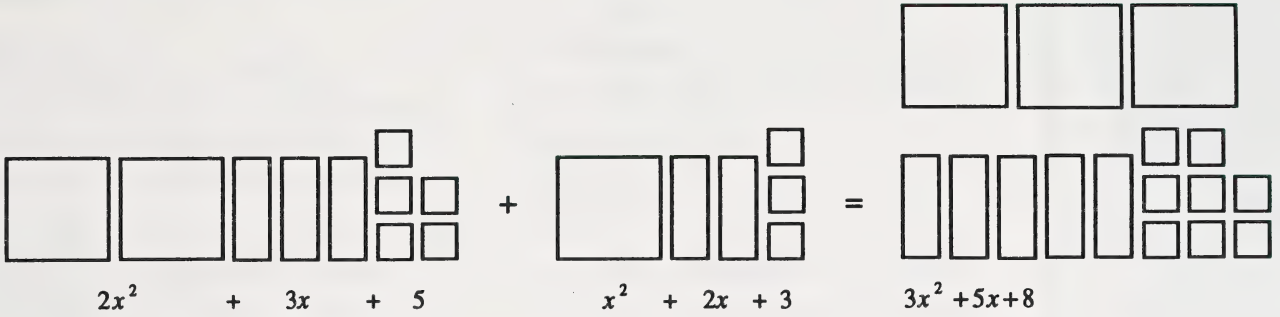
Add $(2x^2+3x+5)+(x^2+2x+3)$.

Solution:

First choose the tiles which will represent each of the polynomials.

Then collect the like tiles and put them together as shown.

$(2x^2+3x+5)+(x^2+2x+3)$



Therefore, $(2x^2+3x+5)+(x^2+2x+3)=3x^2+5x+8$.

Use your polynomial tiles to do at least two of the following questions.

3. $(x^2+2x+1)+(2x^2+3)$

4. $(2x^2+x)+(3x+2)$

5. $x^2+2x+3+2x^2+3x$

For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

You have been adding tiles that are the same.

You cannot add x^2 tiles to x tiles and represent the sum as a single term. The same principle applies to the addition of variables and polynomials.

Like terms are those which contain the same variables with the same exponents.

You can only add or subtract like terms.

Example 3

- Add $2x+3x$.

  Solution:

  Since $2x$ and $3x$ are like terms, you can add them.

  $$2x+3x=(2+3)x$$
  $$=5x$$

  Notice you add the numerical coefficients 2 and 3 and repeat the literal coefficient x.

- Add $3x^2y+7x^2y$.

  Solution:

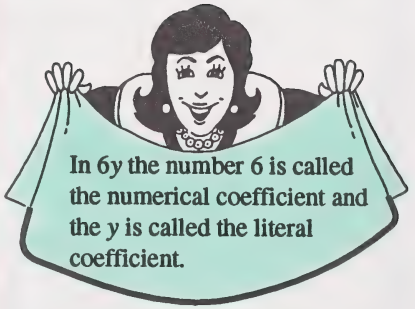
  Since $3x^2y$ and $7x^2y$ are like terms, they can be added.

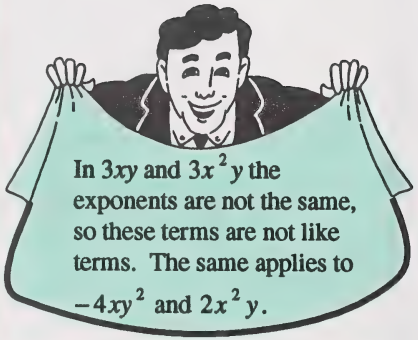
  $$3x^2y+7x^2y=(3+7)x^2y$$
  $$=10x^2y$$

- Add $3xy+3x^2y$.

  Solution:

  The terms $3xy$ and $3x^2y$ are **not** like terms. Therefore, you **cannot simplify by adding**.





If a term does not show a numerical coefficient, the coefficient is understood to be 1.

For example, x is the same as $1x$, and $-y$ is the same as $-1y$.

Do at least three of the questions that follow.

Simplify the following by adding.

6. $5x+3x+7x$

7. $7y^2+2y^2+6y$

8. $5x^2y+3xy^2+3x^2y+2xy^2$

9. $6z^3y+4z^2y+2z^3y+5z^2y$

For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

By now you must be wondering about negatives and subtraction. How are you going to deal with these situations? Read on!

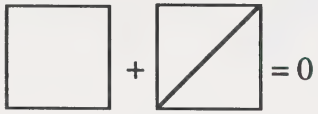
Notice that the **back** of each of your tiles has a diagonal line across it. These lines indicate a **negative** x^2, a negative x, or a negative 1.

This tile represents $-x^2$. It is called a negative flat.

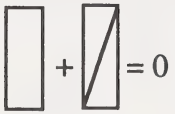
This tile represents $-x$. It is called a negative strip.

This tile represents -1. It is called a negative unit.

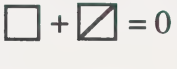
A very important idea is that $1-1=0$ or $1+(-1)=0$. This is called the zero or additive inverses property. In the case of tiles the zero property can be shown in several ways. Study the following sketches.



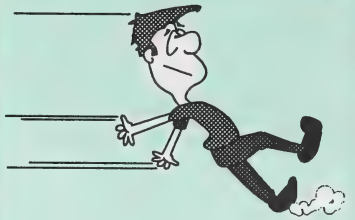
$x^2+(-x^2)=0$



$x+(-x)=0$



$1+(-1)=0$



Stop!

Before studying negative tiles and subtraction, be sure that you understand positive tiles and addition.

Notice how you would represent $x^2 - 2x - 1$ using tiles.

Now try the following questions to see if you understand what to do when you have positive and negative terms in a polynomial. Do at least four of the five questions.

Using the polynomial tiles, represent the following.

10. $-2x^2 + x - 3$

11. $x^2 - 3x - 2$

12. $-3x^2 - 5$

13. $-x^2 + 2x - 1$

14. $-2x^2 - 4x + 3$

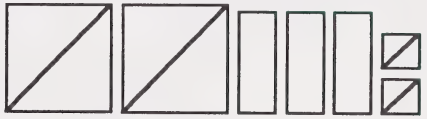
For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

Now you are going to add polynomials which include negative terms.

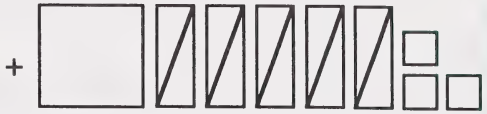
Example 4

Add $(-2x^2 + 3x - 2) + (x^2 - 5x + 3)$.

Solution:



$-2x^2 \quad + \quad 3x \quad - 2$



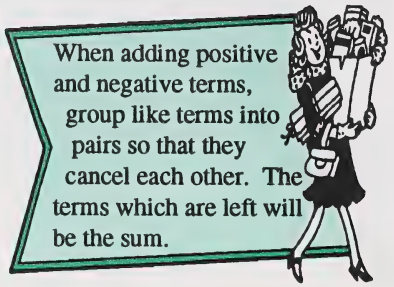
$+ \quad x^2 \quad - \quad 5x \quad + \quad 3$

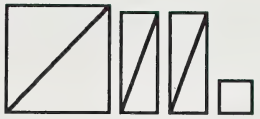
Using the pattern outlined as follows, remove those tiles that add to zero.

Since $-x^2 + x^2 = 0$, remove one negative x^2 tile and the only positive x^2 tile.

Since $-3x + 3x = 0$, remove three negative x tiles and three positive x tiles.

Since $2 + (-2) = 0$, remove two positive unit tiles and two negative unit tiles.


When adding positive and negative terms, group like terms into pairs so that they cancel each other. The terms which are left will be the sum.

You are now left with 

which represents $-x^2-2x+1$.

Therefore, $(-2x^2+3x-2)+(x^2-5x+3)=-x^2-2x+1$.

In order to subtract, you apply the rule of changing the subtraction sign to an addition sign and then changing the sign before the **second** term (the term being subtracted) to its opposite.

This is what happens.

- $6-5$ becomes $6+-5$
- $a-b$ becomes $a+-b$

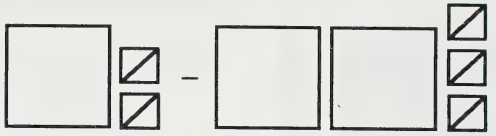
Note that to subtract 5 from 6 which is $6-5$, you change the – sign to a + sign and change 5 to -5. If the number to be subtracted were -5, it would change to 5. In symbol form $6-5$ becomes $6+(-5)$ and $6-(-5)$ becomes $6+5$.

Example 5

Simplify $(x^2-2)-(2x^2-3)$.

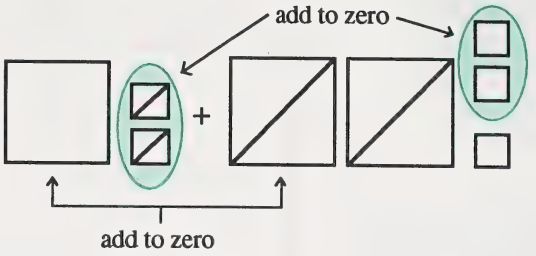
Solution:

First use your tiles to build each polynomial.

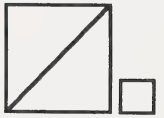


(x^2-2) – $(2x^2-3)$

Then flip over all the tiles which are being subtracted and add.



Now remove those that add to zero, and you get the following:

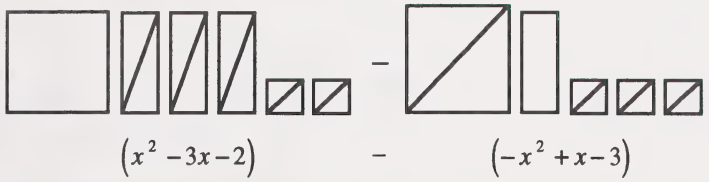


$\therefore (x^2-2)-(2x^2-3)=-x^2+1$

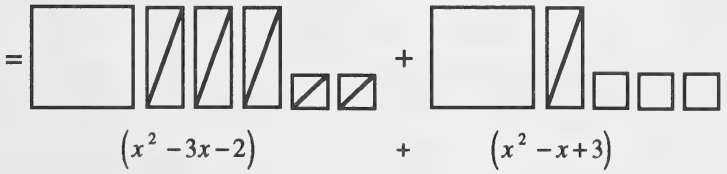
Example 6

Simplify $(x^2 - 3x - 2) - (-x^2 + x - 3)$.

Solution:

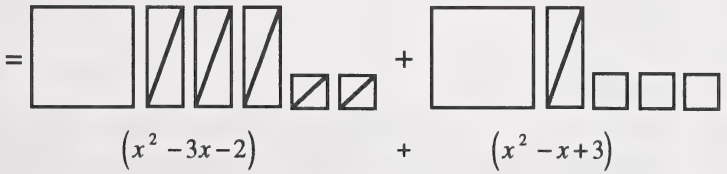


$(x^2 - 3x - 2) \quad - \quad (-x^2 + x - 3)$



$(x^2 - 3x - 2) \quad + \quad (x^2 - x + 3)$

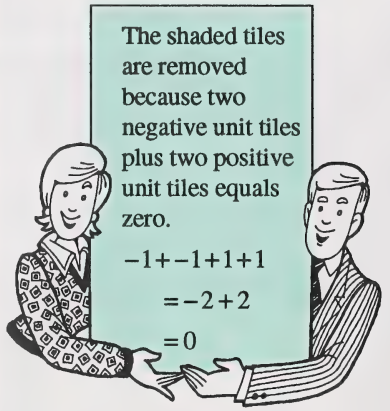
(The tiles being subtracted are flipped over and the operation is changed to addition.)



$(x^2 - 3x - 2) \quad + \quad (x^2 - x + 3)$

(The tiles which are additive inverses are shaded.)

Therefore, $(x^2 - 3x - 2) - (-x^2 + x - 3) = 2x^2 - 4x + 1$.



You have used positive and negative tiles to add and subtract polynomials. Are you ready to apply these operations to polynomials without tiles?

When adding or subtracting polynomials, you often have to work with brackets. Before simplifying you must remove the brackets.

Example 7

Add $(2x+3y)+(-3x+2y)$.

Solution:

$$\begin{aligned}(2x+3y)+(-3x+2y) &= 2x+3y-3x+2y && \text{(Remove brackets.)}\\ &= 2x-3x+3y+2y && \text{(Collect like terms.)}\\ &= (2-3)x+(3+2)y && \text{(Add the numerical coefficients.)}\\ &= -x+5y && \text{(This is final simplified form.)}\end{aligned}$$

Note that $2x+3y-3x+2y$ can be changed to $2x-3x+3y+2y$ because addition is commutative. Saying that addition is commutative means that you can add in any order that you choose.

Example 8

Subtract $(5x^2-3y)-(4x^2-2y+x)$.

Solution:

$$\begin{aligned}(5x^2-3y)-(4x^2-2y+x) &= 5x^2-3y-4x^2+2y-x\\ &= 5x^2-4x^2-3y+2y-x\\ &= (5-4)x^2+(-3+2)y-x\\ &= x^2+(-1)y-x\\ &= x^2-y-x\end{aligned}$$

Change **all** signs inside the bracket when you remove a bracket which has a negative sign in front of it. Notice how $-(4x^2-2y+x)$ becomes $-4x^2+2y-x$.

When you have a **negative sign in front of a bracket**, you must change all of the signs within the bracket when the bracket is removed.

Now it is time to do some problems on your own.

15. Simplify the following using your polynomial tiles.

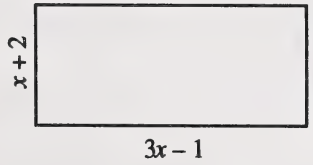
    a. $(-3x^2+5x-4)-(-x^2+3x-6)$

    b. $(2x^2-3x+2)+(-x^2+4x-3)$

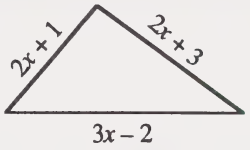
    c. $(x^2+5)-(6-x^2)$

    d. $(3x^2+2x)-(2x^2-x+4)$

    e. Find the perimeter of the rectangle.



    f. Find the perimeter of the triangle.



16. Simplify by removing the brackets and collecting like terms. Do at least three.

    a. $(3x^2-2x)+(4x-x^2)$

    b. $(4y^3x-3y^2x^2)-(2x^2+3y^3x-4y^2x^2)$

    c. $2x^2-(3y^2x^2-4yx^2-5x^2)$

    d. $3x^2y-5xy^2+3y^3-(-3x^2y+2xy^2-2y^3)$

For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

It is important to know that $6-x^2$ means the same as $-x^2+6$ since addition is commutative.

You may have noticed that some of the polynomial expressions seemed mixed up as compared to others which seemed to be in order. There is a general style of writing polynomials which makes it easier to read, understand, and solve polynomial problems. From now on you should write polynomials **alphabetically and in descending order of powers.**

The steps for writing polynomial expressions in alphabetical and descending order are as follows.

Step 1: Write the variables in each term in alphabetical order.

For example,

$5y^2xz - 6zx^2y + 2zy - 3x^3 + 4y^3z$
is rewritten as
$5xy^2z - 6x^2yz + 2yz - 3x^3 + 4y^3z.$

Step 2: Select the terms that contain the variable which occurs first in the alphabet and write these terms first.

For example,

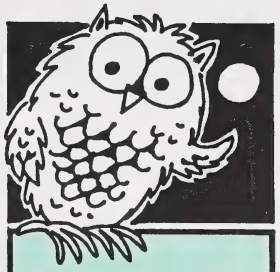
$5xy^2z - 6x^2yz + 2yz - 3x^3 + 4y^3z$
is rewritten as
$5xy^2z - 6x^2yz - 3x^3 + 2yz + 4y^3z.$

Since x is alphabetically first of the variables x, y, and z, you write all the terms containing the variable x first.

Step 3: Rewrite the terms with respect to **descending order for the exponents** or descending order of powers.

For example,

$5xy^2z - 6x^2yz - 3x^3 + 2yz + 4y^3z$
is rewritten as
$-3x^3 - 6x^2yz + 5xy^2z + 4y^3z + 2yz.$



Descending order of powers means arranging exponents from highest to lowest. Try to see why $3x^3 + x^2 + x$ is in descending order of powers.

Alphabetical order is according to the natural order of $a, b, c, d, \ldots, z$. Alphabetical order can be ascending or descending.

The term $5y^2xz$ becomes $5xy^2z$. Notice that the exponent of y^2 is carried with it as you rewrite the term. The term $5y^2xz$ cannot be rewritten as $5x^2yz$.

It is also useful to learn to add or subtract polynomials vertically.

Example 9

Add x^3+3x^2-4x+5 and $2x^3-5x^2+2x-7$ using the vertical method.

Solution:

$$\begin{array}{r} x^3+3x^2-4x+5 \\ 2x^3-5x^2+2x-7 \\ \hline 3x^3-2x^2-2x-2 \end{array}$$

Example 10

Subtract the two polynomials given using the vertical method.

$\left(3x^2y-7xy^2+3y^3\right)-\left(2y^3+2x^2y-5xy^2\right)$

Solution:

Change all the signs in the polynomial being subtracted so that you can add. Then rearrange it alphabetically and in descending order of powers. Rewrite the problem in vertical form. When adding or subtracting polynomials vertically, you must line up each pair of like terms.

This is why $-2x^2y$ is written under $3x^2y$, $5xy^2$ is written under $-7xy^2$, and $-2y^3$ is written under $3y^3$.

$$\begin{array}{r} 3x^2y-7xy^2+3y^3 \\ +\quad -2x^2y+5xy^2-2y^3 \\ \hline x^2y-2xy^2+y^3 \end{array}$$

Example 11

Subtract the two polynomials using the vertical method.

$\left(2x^3y-3x^2+5xy^3\right)-\left(3xy^3-4x^3y\right)$

Solution:

$$\begin{array}{r} 2x^3y-3x^2+5xy^3 \\ +\quad +4x^3y \qquad -3xy^3 \\ \hline 6x^3y-3x^2+2xy^3 \end{array}$$

Notice that there is no term written below $-3x^2$. You could have written $0x^2$ to hold the missing place in the second polynomial making it $4x^3y+0x^2-3xy^3$.

To subtract something means to add its additive inverse. For example, $a-b$ becomes $a+(-b)$ and $-4-7$ becomes $-4+(-7)$.

Now practise what you have learned by doing the following questions.

17. Simplify using the vertical method and write your answers in alphabetical order and in descending order of powers.

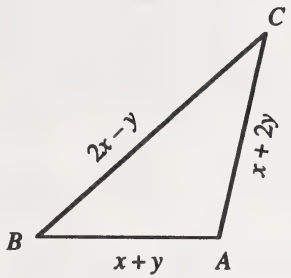
a. $(4x^2-5x+3)-(6x-5x^2+8)$

b. $(x^2-2xy-y^2)-(4y^2-8xy-3x^2)$

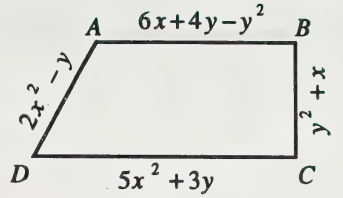
c. $(2x^3-3x^2)+(4x^2-3x^3)-(4x^3-5x^2+3x)$

18. Find the perimeter of one of the figures.

a.



b.

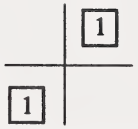


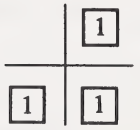
For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

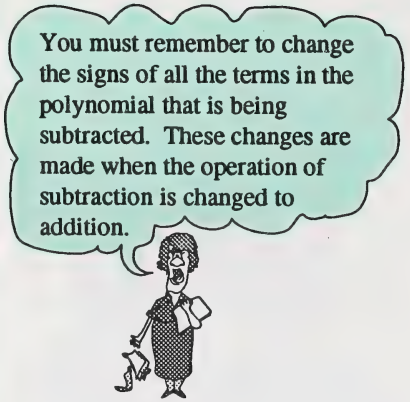
Activity 2

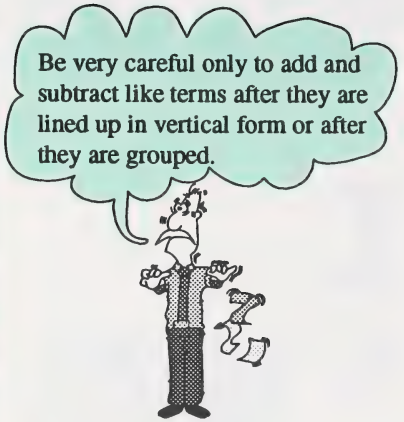
Multiply polynomials by monomials.

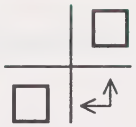
Multiplication of terms can be represented using polynomial tiles.

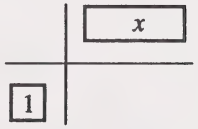
Let  represent 1×1.

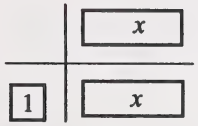
Then  represents $1\times1=1$.

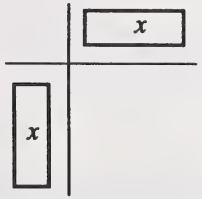

You must remember to change the signs of all the terms in the polynomial that is being subtracted. These changes are made when the operation of subtraction is changed to addition.

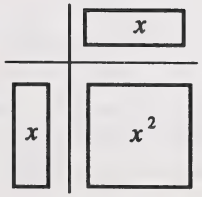


This is because  gives .

Let  represent $1 \times x$.

Then  represents $1 \times x = x$.

Let  represent $x \times x$.

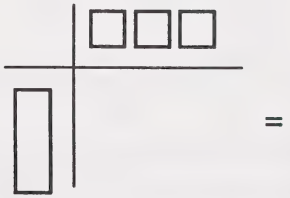
Then  represents $x \times x = x^2$.

Any future use of these or similar diagrams will not have the values shown in the tiles.

Example 12

Represent $x(3)$ using tiles.

Solution:



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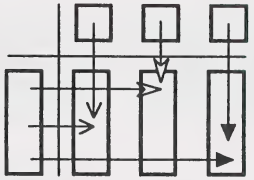
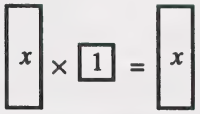
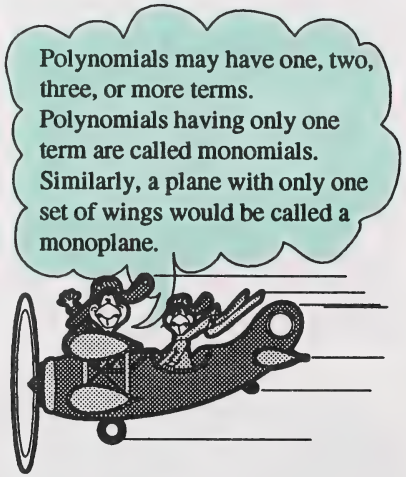


Remember that  .

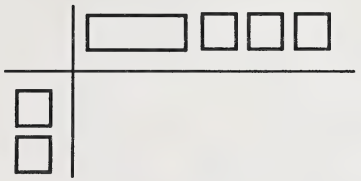
$\therefore\ x(3) = 3x$


Polynomials may have one, two, three, or more terms. Polynomials having only one term are called monomials. Similarly, a plane with only one set of wings would be called a monoplane.

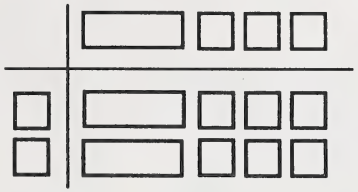
Example 13

Represent $2(x+3)$ using polynomial tiles.

Solution:



Make this representation yourself using polynomial tiles and fill in the actual product. You should get this arrangement.



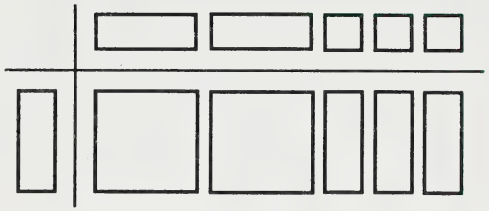
Remember that $\boxed{1} \times \boxed{1} = \boxed{1}$.

Write the product by counting the tiles. You should get $2(x+3)=2x+6$ which means that after the multiplication there are two strips and six units.

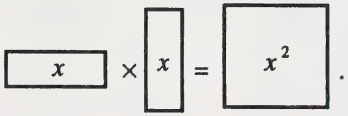
Example 14

Simplify $x(2x+3)$ using polynomial tiles.

Solution:



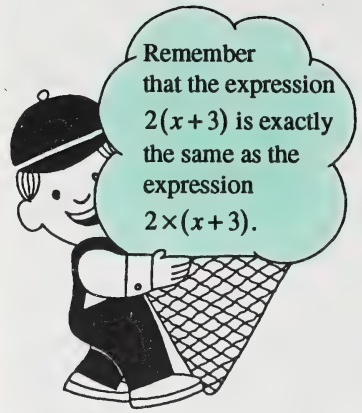
Remember that



Therefore, $x(2x+3)=2x^2+3x$.

Note that $2x^2+3x$ means that you end up with two flats and three strips after the multiplication is completed.

Do at least three of the following questions.

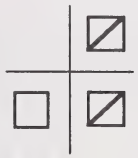


Multiply the following using polynomial tiles.

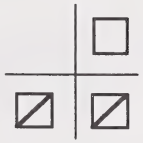
1. $2x(3x+2)$
2. $3(2x+1)$
3. $(x+3)(x)$
4. $3x(x+2)$
5. $x(2x+3)$

For solutions to **Activity 2**, turn to **Appendix A, Topic 3**.

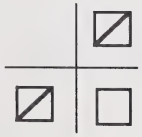
What about those troublesome negatives? Use the same sign rules for multiplication as before. Go over each of the following carefully to ensure you understand everything before going on to study the examples.



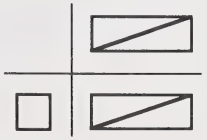
$(1)(-1)=-1$



$(-1)(1)=-1$

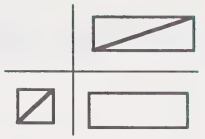


$(-1)(-1)=1$



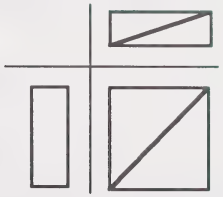
$(1)(-x)=-x$

$(-x)(1)=-x$

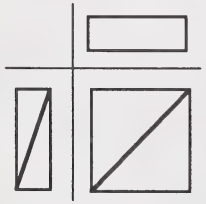


$(-1)(-x)=x$

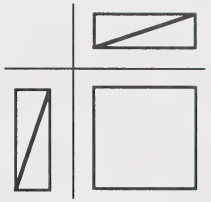
$(-x)(-1)=x$



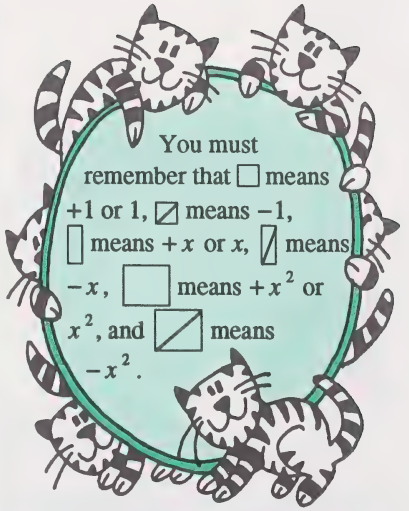
$(x)(-x)=-x^2$



$(-x)(x)=-x^2$



$(-x)(-x)=x^2$



To make sure that you do not make any serious errors, check your understanding of these tile combinations using this simple exercise. Using a sheet of paper, cover $(1)(-1)=-1$.

Look at .

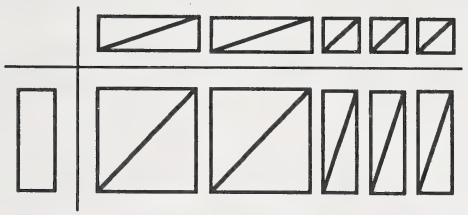
Write down what you think this means. Does your answer match $(1)(-1)=-1$? Go through the other combinations in a similar way before going on to study the examples.

Now study these examples carefully.

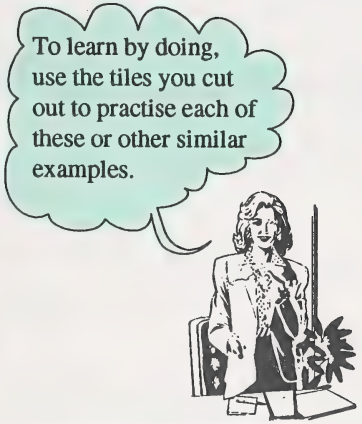
Example 15

Use tiles to change $x(-2x-3)$ to a simplified product.

Solution:



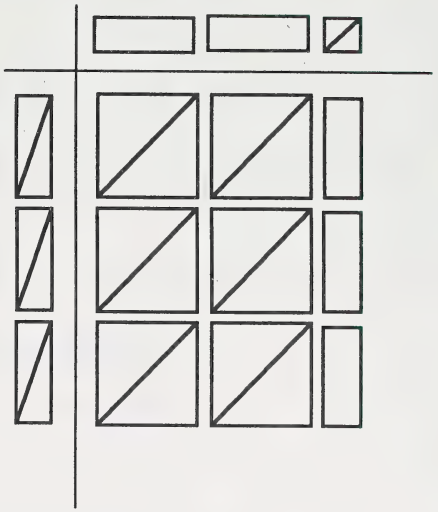
Therefore, $x(-2x-3)=-2x^2-3x$.



Example 16

Multiply $(-3x)(2x-1)$ using tiles.

Solution:



Therefore, $(-3x)(2x-1)=-6x^2+3x$.

Do any three of the questions that follow.

Simplify each of the following using tiles.

6. $(3x-4)(-x)$

7. $(-2x)(-2x-2)$

8. $-2(x-1)$

9. $-3x(-x+2)$

10. $3(2x-1)+x(x-3)$

For solutions to **Activity 2**, turn to **Appendix A, Topic 3**.

Polynomial tiles are useful as long as you do not have expressions involving exponents greater than 2. Algebraic methods are required when exponents are greater than 2.

Recall the Product Law which states that $x^a x^b = x^{a+b}$.

Study these examples.

- $\left(6x^2\right)\left(x^3\right) = 6\left(x^2\right)\left(x^3\right)$
  $= 6\left(x^{2+3}\right)$
  $= 6x^5$

- $\left(3x^5\right)\left(2x^2\right) = (3)(2)\left(x^5\right)\left(x^2\right)$
  $= 6\left(x^{5+2}\right)$
  $= 6x^7$

- $\left(5x^3\right)\left(-3x^2\right) = (5)(-3)\left(x^3\right)\left(x^2\right)$
  $= -15\left(x^{3+2}\right)$
  $= -15x^5$

Whenever you multiply a polynomial by a monomial, you use the distributive property. The general form for this property is $a(b+c) = ab+ac$. Study the following.

- $x\left(2x^2+3\right) = (x)\left(2x^2\right)+(x)(3)$
  $= 2x^3+3x$

- $2x(3x-2) = (2x)(3x)+(2x)(-2)$
  $= 6x^2-4x$

- $8y^3\left(7x^2y-4xy^2\right) = 56x^2y^4-32xy^5$

- $\left(-2z^2\right)\left(-3z^3y-4z^2y^3\right) = 6z^5y+8z^4y^3$

- $\left(3x^2y^3\right)\left(-2x^3y^{-4}\right) = -6x^5y^{-1}$
  $= \dfrac{-6x^5}{y}$

In the last three examples the middle step has been done mentally.

Sometimes the expanded form of a product is not in order. For example, y^3x^2 is not in alphabetical order. Make it a habit always to write the variables of a term in **alphabetical order**. Thus, you rewrite y^3x^2 as x^2y^3.

Polynomials also sometimes end up out of order. For example, look at $2y^3x+3x^3y-5y^2x^2$.

First rewrite **each term** in alphabetical order.

$2xy^3+3x^3y-5x^2y^2$

Then rewrite the order of the terms so that they are in **descending order** of powers for the exponents which apply to the x-variable.

$3x^3y-5x^2y^2+2xy^3$

Example 17

Write each of the following in alphabetical order and descending order of powers.

- $4y^2x-3y^3x^2-4y+3-4yx^3$

  Solution:

  $$4y^2x-3y^3x^2-4y+3-4yx^3=-4x^3y-3x^2y^3+4xy^2-4y+3$$

- $2-5xy^2z+3yx^2z-2z^2yx^3$

  Solution:

  $$2-5xy^2z+3yx^2z-2z^2yx^3=-2x^3yz^2+3x^2yz-5xy^2z+2$$

The degree of a term is the sum of the exponents of its variables.

Example 18

What is the degree of $3x^2y^3z$?

Solution:

The degree of $3x^2y^3z$ is 6 because the exponents of x^2, y^3, and z are 2, 3, and 1, respectively, and $2 + 3 + 1 = 6$.

The degree of a variable is the exponent of the variable.

Example 19

What is the degree of x in the term $3x^7y^4$.

Solution:

The degree of x is 7.

The degree of a polynomial is the degree of the term which has the highest degree.

Example 20

What is the degree of $3x^5y^4 + 2x^4y^3 + 3$?

Solution:

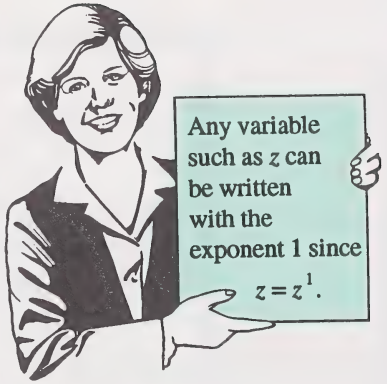
The degree of the given polynomial is 9 because this is the degree of the term which has the highest degree of the three terms. The degrees of the three terms are 9, 7, and 0, respectively.

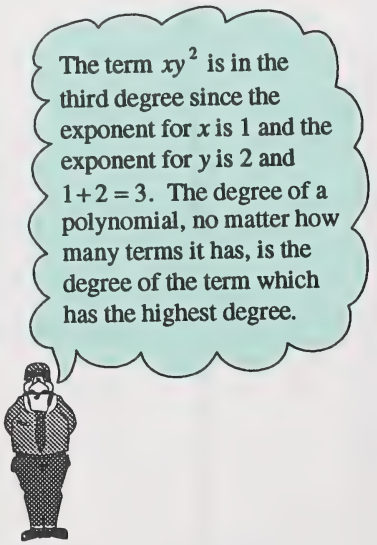
Do questions 11 and 12 and any other two questions.

11. Simplify each of following.

    a. $(3x^2y^3)(2xy^4)$

    b. $(-3x^3y^5)(-2xy^4)(4x^2y)$




The term xy^2 is in the third degree since the exponent for x is 1 and the exponent for y is 2 and $1+2=3$. The degree of a polynomial, no matter how many terms it has, is the degree of the term which has the highest degree.

12. Expand each of the following and write your answers in alphabetical order and descending order of powers.

    a. $2y(3x^2 - y^3)$ b. $-x(y-3x)$

    c. $(-2z)(-3z)(2x^2z - 3xz^3)$

13. What is the degree of the polynomial $6x^3y^2 - 5xy^3 + 6$?

14. What is the degree of $7x^4y^7$?

15. What is the degree of y in the term $8x^7y^6z^4$?

For solutions to **Activity 2**, turn to **Appendix A, Topic 3.**

Activity 3

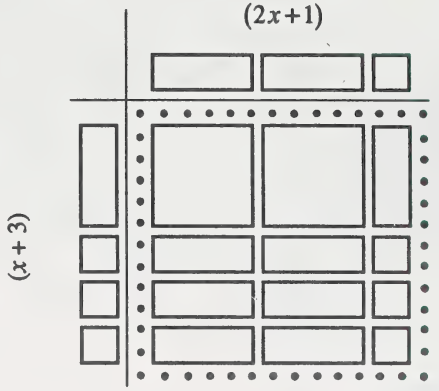
Multiply binomials by binomials.

A binomial is a polynomial which has two terms. Use your polynomial tiles to multiply binomials by binomials.

Example 21

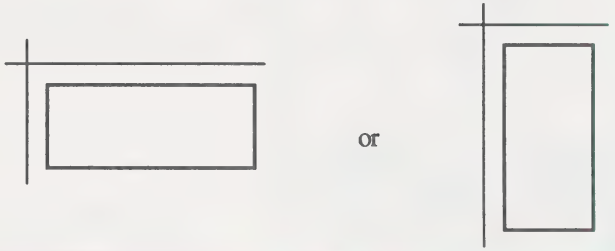
Multiply $(x+3)(2x+1)$ using polynomial tiles.

Solution:



Therefore, $(x+3)(2x+1) = 2x^2 + 7x + 3$.

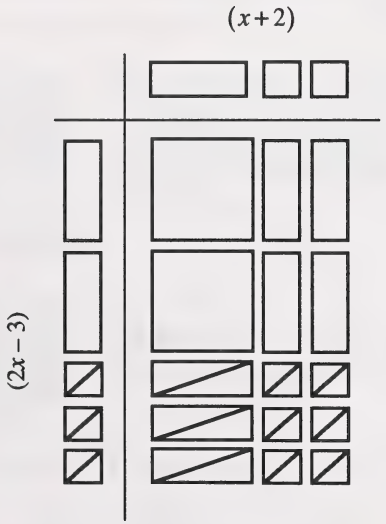
Note that the dotted line in the previous example forms a rectangle. When you are arranging the tiles, you must end up with a pattern that forms a rectangle as shown. If you do not have a complete rectangle, you have made a mistake.



Example 22

Expand or multiply the two binomials $(2x-3)$ and $(x+2)$ using polynomial tiles.

Solution:



The shaded tiles can be paired and cancelled since $-x+x=0$ on three occasions.

The expansion or multiplication yields $2x^2+4x-3x-6$ which can be simplified to $2x^2+x-6$ when the two like terms are combined.

$\therefore (2x-3)(x+2)=2x^2+x-6$

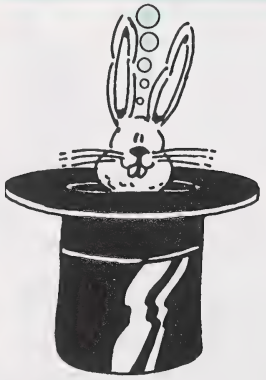
Do any three of the following questions. Do all of them if you feel you need more practice.

Expand or multiply the following binomials using polynomial tiles.

1. $(-x+2)(-2x-1)$
2. $(3x+4)(2x-1)$
3. $(2x-5)(x-2)$
4. $(x+3)(-3x+1)$
5. $(x-4)(3-2x)$

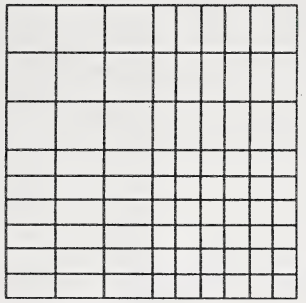
For solutions to **Activity 3**, turn to **Appendix A, Topic 3**.

To expand polynomials means to multiply them to get another polynomial which is a product.



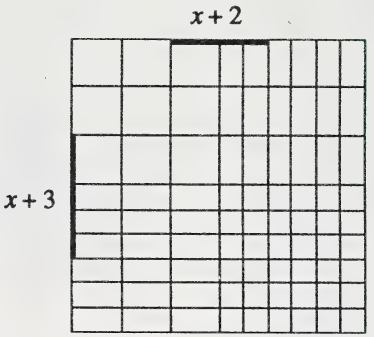
Have you noticed that all of your solutions have a pattern in the shape of a rectangle?

Another method similar to using polynomial tiles when expanding or multiplying binomials that does not involve anything more than paper and pencil is the **binomial grid**. Here is what a binomial grid looks like.

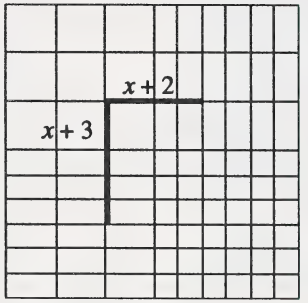


Notice that the individual parts look similar to the products formed when you multiplied using polynomial tiles. You can find a collection of shapes which stand for x^2, some that stand for x, and still others that represent 1. Try representing some products of polynomials so you can see what the grid represents and how it can be used for multiplying binomials. Study how to represent $(x+3)(x+2)$ on a binomial grid.

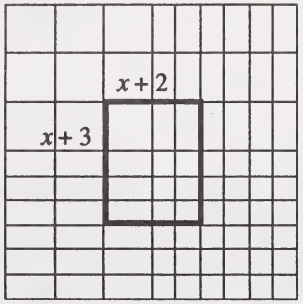
First place $x+3$ along the left side of the grid. Remember the area of the large square is x^2 so each side is x. The area of each strip is x so the narrow side of each strip is 1. Similarly, place $x+2$ at the top of the grid as shown.



Move the two factors together on the grid so that they meet at a common spot at the top left corner.

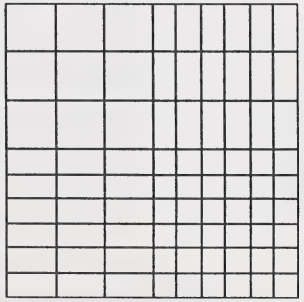


Now complete the rectangle by drawing lines along the other two sides.



Now count and put together the different shapes within the outlined rectangle. You get $x^2 + 5x + 6$ which is the same product that you would get if polynomial tiles were used.

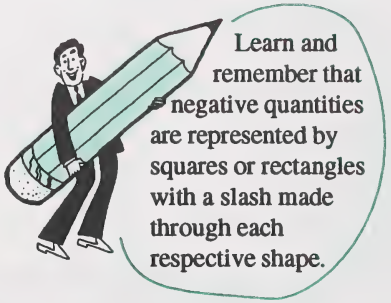
Try showing $(2x+3)(x+5)$ on the binomial grid.



Turn the page to see if you used the correct diagram. The answer is in the right margin. If you used the correct diagram, go on. If you made an error, study the correct diagram and try it again.

What happens if you are asked to represent a negative value on the grid? You make a negative slash through the square or rectangle to get the same forms that were used earlier for negative tiles.

If you want to represent $-x$, draw a slash across one of the rectangles that represents x. The same applies if you want to get a $-x^2$ or a -1.



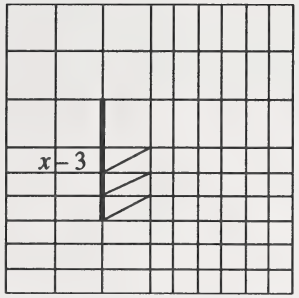
Learn and remember that negative quantities are represented by squares or rectangles with a slash made through each respective shape.

Example 23

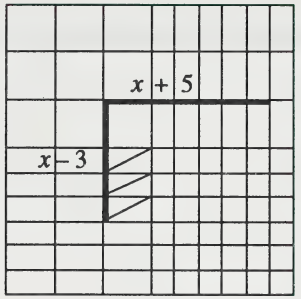
Represent $(x-3)(x+5)$ on a binomial grid.

Solution:

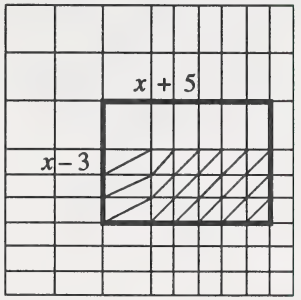
First put a solid line for the $x-3$ factor on the left side of the centre of the grid. Start one x^2 unit up and one x^2 unit left of the centre of the grid since the coefficient of each of the x-terms is one.



Next add the solid line for the $x+5$ factor.



Complete the rectangle and for each of the three rows where the x strip is negative, mark the units negative.

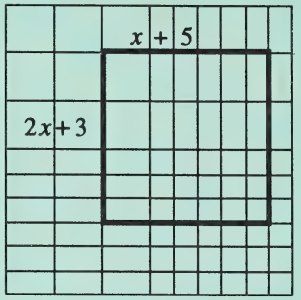


$$\therefore (x-3)(x+5) = x^2 + 5x - 3x - 15$$

$$= x^2 + 2x - 15$$

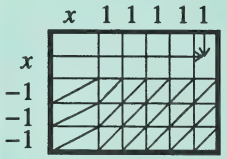
Notice that you cannot remove the rectangles that cancel to zero as you do with polynomial tiles. You must write down the total of each shape and then add to simplify.

This is the solution to the problem on the previous page.



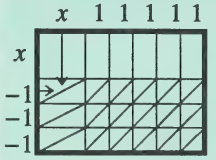
$$\therefore (2x+3)(x+5) = 2x^2 + 13x + 15$$

This diagram shows how multiplication is carried through to get x.



$x \times 1 = x$

This diagram shows how multiplication is carried through to get $-x$.



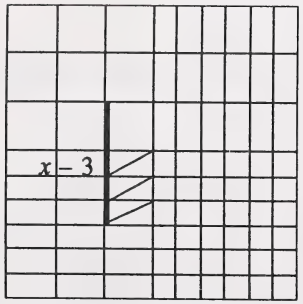
$x \times (-1) = -x$

Example 24

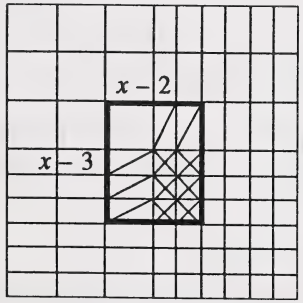
Expand $(x-3)(x-2)$ using the binomial grid.

Solution:

First work with $(x-3)$ using one x^2 and three x strips. Make the x strips negative since the 3 is negative in $(x-3)$.



Now draw the value $(x-2)$ and complete the rectangle. If you make an already negative value negative, draw a slash in the opposite direction.



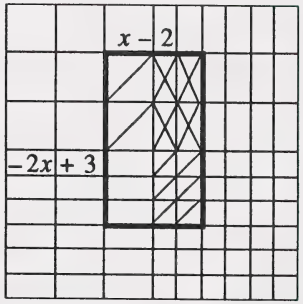
When you have an area or shape that is crossed out (☒), the quantity is positive.

$$\therefore (x-3)(x-2) = x^2 - 3x - 2x + 6$$
$$= x^2 - 5x + 6$$

Example 25

Expand $(-2x+3)(x-2)$.

Solution:

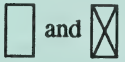


This represents the following:

$$(-2x+3)(x-2) = -2x^2 + 3x + 4x - 6$$
$$= -2x^2 + 7x - 6$$

Now try some yourself.

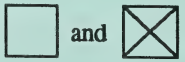
It is important to learn and remember the following:

-  and

  These both mean x or $+x$.

- ☐ and ☒

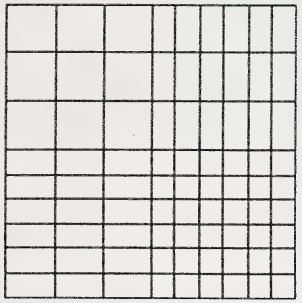
  These both mean 1 or +1.

-  and

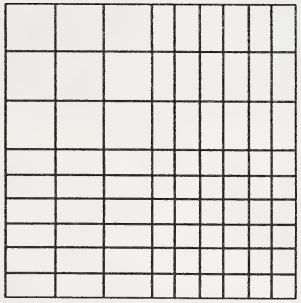
  These both mean x^2 or $+x^2$.

Expand any three using the binomial grid.

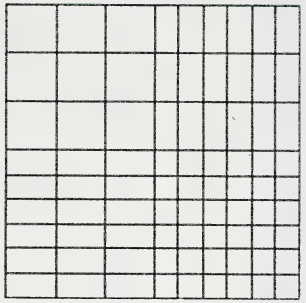
6. $(3x-2)(-x+1)$



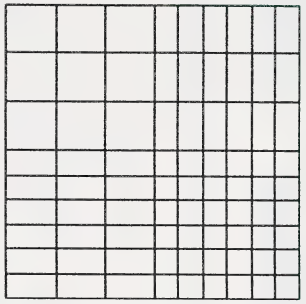
7. $(-2x-2)(x-3)$



8. $(2x+1)(x+3)$



9. $(3x+4)(2x+3)$



For solutions to **Activity 3**, turn to **Appendix A, Topic 3.**

Another method for simplifying binomials by expansion is by using the **FOIL** method.

You have already learned to multiply a binomial by a monomial.

For example,

- $x(x+1)=x^2+x$
- $2(x+1)=2x+2$

Multiplying a binomial by a binomial is much the same. You simply perform each multiplication and collect like terms.

For example,

$$(x+2)(x+1)=x^2+x+2x+2$$
$$=x^2+3x+2$$

Look at what happens in complete detail.

- Multiply $x(x+1)=x^2+x$.
  ↑ ↑
  first term of first binomial; second binomial

- Multiply $2(x+1)=2x+2$.
  ↑ ↑
  second term of first binomial; second binomial

- Add the two products.
  Add x^2+x and $2x+2$.
  $x^2+x+2x+2=x^2+3x+2$

A shorter form of this method is shown as follows.

$$(\boxed{x}+2)(\boxed{x}+1)=\boxed{x^2}+x+2x+2$$

First terms of each binomial are multiplied to give x^2.

$$(\boxed{x}+2)(x+\boxed{1})=x^2+\boxed{x}+2x+2$$

Outside terms of both binomials are multiplied to give x.

$$(x+\boxed{2})(\boxed{x}+1)=x^2+x+\boxed{2x}+2$$

Inside terms of both binomials are multiplied to give $2x$.

$$(x+\boxed{2})(x+\boxed{1})=x^2+x+2x+\boxed{2}$$

Last terms of both binomials are multiplied to give 2.

After adding the four products, you get
$x^2+x+2x+2=x^2+3x+2$.

The **FOIL** method is used to multiply a binomial by a binomial. This is the pattern which is always followed.
In $(x+2)(x+1)$,

- the **First** terms are x and x
- the **Outside** terms are x and 1
- the **Inside** terms are 2 and x
- the **Last** terms are 2 and 1

This pattern is remembered as **FOIL** according to the first letters of the words **F**irst, **O**utside, **I**nside, and **L**ast.

Remember, you must get four product terms since you multiply a total of four times.

The middle two terms of the product can usually be added so the final answer will have a total of three terms. These three terms result in a trinomial product.

Examine in detail one more multiplication problem involving binomials.

Find the product for $(2x-1)(x+3)$.

Using **F**, $(\boxed{2x}-1)(\boxed{x}+3)$ gives $(2x)(x)=2x^2$.

Using **O**, $(\boxed{2x}-1)(x+\boxed{3})$ gives $(2x)(3)=6x$.

Using **I**, $(2x\boxed{-1})(\boxed{x}+3)$ gives $(-1)(x)=-x$.

Using **L**, $(2x\boxed{-1})(x+\boxed{3})$ gives $(-1)(+3)=-3$.

$$\therefore (2x-1)(x+3)=2x^2+6x-x-3$$
$$=2x^2+5x-3$$

Try the following questions to become confident using the **FOIL** method of expansion.

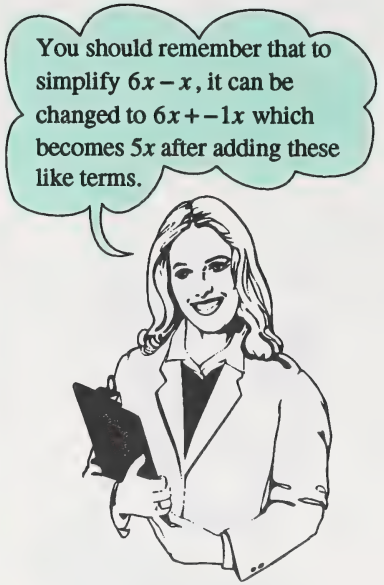
Do at least eight of the following questions.

Expand and simplify. Write all products in descending order of powers.

10. $(x+2)(x+2)$
11. $(y+3)(y+3)$
12. $(2x+1)(x-3)$
13. $(x+2)(3x-5)$
14. $(3-x)(x+4)$
15. $(x-1)(x-1)$
16. $(2-y)(2-y)$
17. $(x+2)(x-2)$
18. $(3-y)(3+y)$
19. $(2x-1)(2x+1)$
20. $(3x+2y)(2x-3y)$
21. $(6y-8x)(4y-7x)$

For solutions to **Activity 3**, turn to **Appendix A, Topic 3**.

If you have access to a videocassette player, you may view the video titled *Multiplying Polynomials*. This program will help you review Activities 2 and 3 of this topic.



Activity 4

Divide polynomials by monomials.

Recall the Quotient Law for powers which states that $\frac{x^a}{x^b} = x^{a-b}$.

You can also use the Quotient Law for powers to simplify the division of a monomial by a monomial. Divide the coefficients of the terms. Do the numerical coefficients and the literal coefficients separately.

Look at these examples.

- $\frac{x^3}{x^2} = x^{3-2}$
  $= x$

- $\frac{9x^4}{3x} = 3x^{4-1}$
  $= 3x^3$

- $\frac{-16x^7}{2x^3} = -8x^{7-3}$
  $= -8x^4$

- $8y^6 + (-4y^2) = -2y^{6-2}$
  $= -2y^4$

Division of polynomials by monomials is much the same. Simply divide **each term** in the polynomial by the monomial.

Example 26

- Simplify $\frac{2x^2 + 4x}{2}$.

  Solution:

  $$\frac{2x^2 + 4x}{2} = \frac{2x^2}{2} + \frac{4x}{2}$$
  $$= x^2 + 2x$$

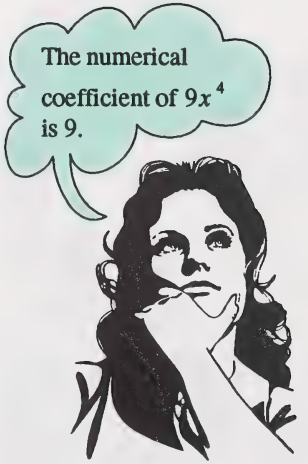
  Notice that **both** $2x^2$ and $4x$ are divided by 2.

- Divide $x^2 + 3x$ by x.

  Solution:

  $$\frac{x^2 + 3x}{x} = \frac{x^2}{x} + \frac{3x}{x}$$
  $$= x + 3$$

  Notice that **both** x^2 and $3x$ are divided by x.



- Simplify $\frac{3x^2+6x}{3x}$.

Solution:

To simplify this, you must divide.

$$\frac{3x^2+6x}{3x}=\frac{3x^2}{3x}+\frac{6x}{3x}$$
$$=x+2$$

- Divide $4a^2x+8ax^2$ by $2ax$.

Solution:

$$\frac{4a^2x+8ax^2}{2ax}=\frac{4a^2x}{2ax}+\frac{8ax^2}{2ax}$$
$$=2a+4x$$

- Simplify $\frac{15x^3y^2z-9x^2y^3z^2}{3x^2y}$.

Solution:

This is simplified using division.

$$\frac{15x^3y^2z-9x^2y^3z^2}{3x^2y}=\frac{15x^3y^2z}{3x^2y}-\frac{9x^2y^3z^2}{3x^2y}$$
$$=5xyz-3y^2z^2$$

Try the following questions.

1. Simplify each of the following expressions using division. Remember to divide the denominator into each term of the numerator.

   a. $\frac{4x-2}{2}$

   b. $\frac{27x^3y+9xy^3}{3xy}$

   c. $\frac{14a^2b-7ab^2+21b^3}{7b}$

   d. $\frac{6x^3y-9x^2y^2+12xy^3}{-3xy}$

2. The area of a rectangular table top is $6x^2+4xy$ square units. If the width of this table is $2x$ units, find the length of the table. Use this diagram to see what must be done.

$A=6x^2+4xy$ $\quad w=2x$

$l=?$

For solutions to **Activity 4**, turn to **Appendix A, Topic 3**.

Check your division by using multiplication. Simplify this expression by division.

$$\frac{3x^2+6x}{3x}=\frac{3x^2}{3x}+\frac{6x}{3x}$$
$$=x+2$$

To check, use multiplication as shown here.

$$3x(x+2)=3x(x)+3x(2)$$
$$=3x^2+6x$$

Can you divide polynomials by monomials using the binomial grid? Yes, you can! If you mastered the multiplication of binomials through the use of a binomial grid, study the following examples which show division using a binomial grid. Then complete the questions that follow.

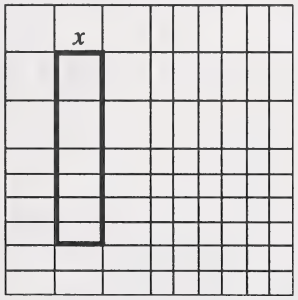
Example 27

Divide $2x^2 + 4x$ by x using a binomial grid.

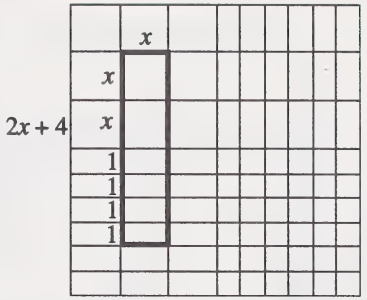
Solution:

Draw a rectangle on the grid with the **area** $2x^2 + 4x$, using x as the length of one side.

The area of the rectangle is $2x^2 + 4x$.



Now determine the length of the other side of the rectangle.



Recall that each side of the large square is x and the width of the x strip is one unit.

$$\therefore \frac{2x^2 + 4x}{x} = \frac{2x^2}{x} + \frac{4x}{x}$$
$$= 2x + 4$$

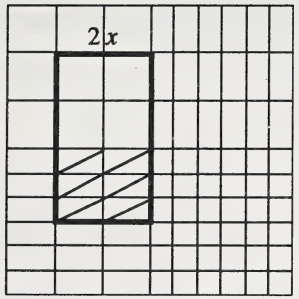
Example 28

Divide $4x^2 - 6x$ by $2x$ using the binomial grid.

Solution:

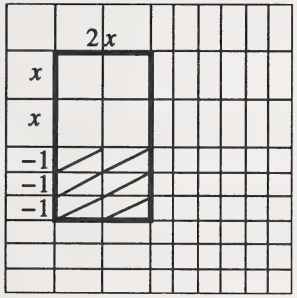
Draw a rectangle on the grid with an area of $4x^2 - 6x$, using $2x$ as the length of one side.

The area of the rectangle is $4x^2 - 6x$.



Now determine the length of the other side of the rectangle.

$2x-3$



$$\therefore 4x^2 - 6x + 2x = \frac{4x^2}{2x} - \frac{6x}{2x}$$
$$= 2x - 3$$

Remember that division can be checked by multiplication. For example, $6 \div 3 = 2$ and so $3 \times 2 = 6$.

When checking division by multiplication, you use the quotient and the divisor when multiplying. For example, in $24 \div 6 = 4$ the quotient is 4 and the divisor is 6. To check, you use $4 \times 6 = 24$. Because the product 24 is the same as the dividend 24, the division is correct.

Check that $4x^2 - 6x + 2x = 2x - 3$.

Since $2x(2x-3) = 4x^2 - 6x$, you know that the division was correct.

Try checking your division by multiplying.

Do these questions.

Divide using the binomial grid. Check your division by multiplying.

3. $\dfrac{4x^2 + 8x}{2x}$

4. $\dfrac{6x^2 + 9x}{3x}$

5. $\dfrac{2x^2 - 4x}{2x}$

For solutions to **Activity 4**, turn to **Appendix A, Topic 3**.

Remember that division can be show in three different ways. Notice where the dividend, the divisor, and the quotient occur in each case.

divisor
↓
• $24 \div 6 = 4$ ← quotient
↑
dividend

dividend
↓
• $\dfrac{24}{6} = 4$ ← quotient
↑
divisor

4 ← quotient
• $6\overline{)24}$ ← dividend
↑
divisor

Notice how this division is changed to multiplication.

quotient
↓
$6 \times 4 = 24$ ← dividend
↑
divisor

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Why is multiplication of polynomials so difficult for some students and so easy for other students? This question could well be impossible to answer, but if you are a student who has difficulty with algebraic multiplication, this section could be useful and helpful.

- Do you have problems with simple multiplication of integers? Try these.

  $(-5)(3) =$

  $(-8)(-7) =$

  If you had trouble getting -15 and 56 for the answers, you should go back to Unit 1 and review the operations involving integers. These skills are absolutely essential for further work in mathematics.

- Do you have problems with operations involving powers? Try these.

  $(x)(x^2) =$

  $(yb)(y^2 b) =$

  If you had trouble getting x^3 and $y^3 b^2$, you have to master the skills from Topic 2 of this unit. Here again, these ideas and concepts are essential building blocks for all algebra work.

If you do not have trouble with the ideas and skills previously described, you should be able to master polynomial multiplication.

Remember that for **every** multiplication of one term by another you must have one term in the product.

Example 29

Multiply $3(4+5)$.

Solution:

$$\underbrace{3(4+5)} = \underbrace{(3\times 4)+(3\times 5)}$$

$$= 12+15$$

$$= 27$$

Two multiplications yield two terms. In this case 12 and 15 are combined to become 27.

If you multiply a trinomial by a monomial, you get three terms in the product.

Example 30

Expand $2(x^2+x+1)$.

Solution:

$$2(x^2+x+1)=2(x^2)+2(x)+2(1)$$
$$=2x^2+2x+2$$

How many terms must you have in the product, before you can possibly collect like terms, when you multiply a binomial by a binomial? Look at the following example.

Example 31

Expand $(x+2)(x+4)$.

Solution:

$$(x+2)(x+4)=x^2+4x+2x+8$$

Note that there are four terms to start. If you collect like terms, you get x^2+6x+8.

How many terms must you have in the product, before collecting like terms, when you multiply a binomial by a trinomial? Look at the next example.

Example 32

Expand $(x+3)(x^2+2x+3)$.

Solution:

There must be six terms in the product before collecting like terms, since the trinomial which has three terms is multiplied by each of the terms in the binomial which has two terms $(3\times 2=6)$. In complete detail, the calculations are as follows:

$$x\times(x^2+2x+3)=x^3+2x^2+3x$$
$$3\times(x^2+2x+3)=3x^2+6x+9$$

Now collect like terms to get the following:

$$x^3+2x^2+3x+3x^2+6x+9$$
$$=x^3+5x^2+9x+9$$

Try some of the questions that follow. To help you, the solutions are shown in complete detail.

Expand and simplify completely. Write your answers in descending order of powers.

1. $(-2)(x+1)$
2. $(x)(x-3)$
3. $x^2(x-2)$
4. $(x+1)(5)$

There are two ways to expand or multiply two binomials. These methods are the vertical method and the FOIL method. The vertical method works as follows:

$$\begin{array}{r} x+2 \\ \underline{x+4} \\ x^2+2x \\ \underline{+4x+8} \\ x^2+6x+8 \end{array}$$

The FOIL method gives the following:

$$(x+2)(x+4)=(x)(x)+(x)(4)$$
$$+(2)(x)+(2)(4)$$
$$=x^2+4x+2x+8$$
$$=x^2+6x+8$$

5. $(2a)(a^2 - 3a)$

6. $(b-2)(b^2 + 3)$

7. $(c+b)(c+b)$

8. $(2a-1)(a+3)$

For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.

Division of polynomials can be done using a long division method. Go through the next examples and then try a few yourself.

Example 33

Simplify $\dfrac{16x^4 - 4x^3 + 8x^2}{2x^2}$.

Solution:

Set up as a long division question.

$$2x^2 \overline{)16x^4 - 4x^3 + 8x^2}$$

Now divide $2x^2$ into the first term of the dividend and write the result above the line and over the first term of the dividend.

$$\begin{array}{r} 8x^2 \\ 2x^2 \overline{)16x^4 - 4x^3 + 8x^2} \end{array}$$

Then multiply the $8x^2$ by $2x^2$ to check your work by subtracting. Notice how this is done.

$$\begin{array}{r} 8x^2 \\ 2x^2 \overline{)16x^4 - 4x^3 + 8x^2} \\ 16x^4 \\ \hline 0 \end{array}$$

Continue by dividing $2x^2$ into the next term.

$$\begin{array}{r} 8x^2 - 2x \\ 2x^2 \overline{)16x^4 - 4x^3 + 8x^2} \end{array}$$

Multiply $-2x$ by $2x^2$ and check by subtraction as shown.

$$\begin{array}{r} 8x^2 - 2x \\ 2x^2 \overline{)16x^4 - 4x^3 + 8x^2} \\ -4x^3 \\ \hline 0 \end{array}$$

Do the last division and check by subtraction one more time.

$$\begin{array}{r} 8x^2 - 2x + 4 \\ 2x^2 \overline{)16x^4 - 4x^3 + 8x^2} \\ +8x^2 \\ \hline 0 \end{array}$$

The final answer is $8x^2 - 2x + 4$.

Check:

$$\begin{array}{r} 8x^2 - 2x + 4 \\ \times \quad 2x^2 \\ \hline 16x^4 - 4x^3 + 8x^2 \end{array}$$

Example 34

Divide $\left(12x^3y^2 - 6x^2y^3 + 9xy^4\right)$ by $3xy^2$.

Solution:

Step 1:

$$3xy^2 \overline{)\,12x^3y^2 - 6x^2y^3 + 9xy^4}$$

$4x^2$

$12x^3y^2$

0

Step 2:

$$3xy^2 \overline{)\,12x^3y^2 - 6x^2y^3 + 9xy^4}$$

$4x^2 \quad -2xy$

$-6x^2y^3$

0

Step 3:

$$3xy^2 \overline{)\,12x^3y^2 - 6x^2y^3 + 9xy^4}$$

$4x^2 \quad -2xy \quad +3y^2$

$+9xy^4$

0

$$\therefore \frac{12x^3y^2 - 6x^2y^3 + 9xy^4}{3xy^2} = 4x^2 - 2xy + 3y^2$$

Check:

$$\begin{array}{r} 4x^2 - 2xy + 3y^2 \\ \times \quad 3xy^2 \\ \hline 12x^3y^2 - 6x^2y^3 + 9xy^4 \end{array}$$

Try dividing these using long division. Check your answers.

9. $\left(6x^3 - 9x^2 + 3x\right) \div 3x$

10. $\left(9x^6 - 6x^4 - 18x^2\right) \div \left(-3x^2\right)$

11. $\left(16x^3y^3 - 8x^2y^2 + 4xy\right) \div (2xy)$

For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.

These points are important to remember .

- Additive inverses are two values which add to zero such as $-6x^2 + 6x^2 = 0$.
- To subtract means to add the additive inverse of the value being subtracted as in the following:

$$-6x^2 - 3x^2 = -6x^2 + \left(-3x^2\right)$$
$$= -9x^2$$

In this section you divided polynomials which worked out evenly leaving zero as the remainder.

Extensions

You can extend the idea of the long division of polynomials by monomials to the division of polynomials by polynomials which are not monomials. Go over this example of the division of a trinomial by a binomial.

Example 35

Divide $x^2+13x+42$ by $x+6$.

Solution:

$$x+6\overline{)x^2+13x+42}$$

Divide the **first** term of the trinomial by the **first** term of the divisor and write the answer above the **first** term of the trinomial.

$$\begin{array}{r} (x) \\ (x)+6\overline{)(x^2)+13x+42} \end{array}$$

Here $x^2+x=x$.

Now multiply **both terms** in the binomial by the answer and write the product under the first two terms of the trinomial.

$$\begin{array}{r} (x) \\ (x+6)\overline{)x^2+13x+42} \\ (x^2+6x) \end{array}$$

Here $x(x+6)=x^2+6x$.

Subtract x^2+6x from x^2+13x.

$$\begin{array}{r} x \\ x+6\overline{)x^2+13x+42} \\ \underline{x^2+6x} \\ (+7x) \end{array}$$

Here $(x^2+13x)-(x^2+6x)$ becomes $0+7x$ or $7x$.

Now bring down the third term of the trinomial.

$$\begin{array}{r} x \\ x+6\overline{)x^2+13x+42} \\ \underline{x^2+6x} \\ 7x+(42) \end{array}$$

Divide the first term of $7x+42$ by the first term of the binomial.

$$\begin{array}{r} x\ (+7) \\ (x)+6\overline{)x^2+13x+42} \\ x^2+6x \\ \hline (7x)+42 \end{array}$$

Multiply $7(x+6)$.

$$\begin{array}{r} x\ +(7) \\ (x+6)\overline{)x^2+13x+42} \\ x^2+6x \\ \hline 7x+42 \\ (7x+42) \end{array}$$

Subtract.

$$\begin{array}{r} x\ +7 \\ x+6\overline{)x^2+13x+42} \\ x^2+6x \\ \hline 7x+42 \\ 7x+42 \\ \hline (0) \end{array}$$

Check: $(x+6)(x+7)=x^2+7x+6x+42$

$=x^2+13x+42$

Here is an example of a more complex division.

Example 36

Divide $4x^3-14x^2+14x-3$ by $2x-3$.

Solution:

$$\begin{array}{r} (2x^2) \\ 2x-3\overline{)4x^3-14x^2+14x-3} \end{array}$$

Here $4x^3+2x=(2x^2)$.

$$\begin{array}{r} 2x^2 \\ 2x-3\overline{)4x^3-14x^2+14x-3} \\ (4x^3-6x^2) \\ \hline \end{array}$$

Here $2x^2(2x-3)=(4x^3-6x^2)$.

Stop for a moment to review this important mathematical rule.

Remember that when you remove parentheses which come after a negative sign, the signs of all the terms inside these parentheses change to the opposite of what they were in the first place. What this means is that $-(-2x^2+7x-3)$ would change to $2x^2-7x+3$ when the parentheses are removed.

$$
\begin{array}{r}
2x^2 \\
2x-3\overline{)\,4x^3-14x^2+14x-3} \\
\underline{4x^3-6x^2} \\
-8x^2+14x
\end{array}
$$

Here $\left(4x^3-14x^2\right)-\left(4x^3-6x^2\right)=4x^3-14x^2-4x^3+6x^2$

$=\boxed{-8x^2}$

$$
\begin{array}{r}
2x^2\ \boxed{-4x} \\
2x-3\overline{)\,4x^3-14x^2+14x-3} \\
\underline{4x^3-6x^2} \\
-8x^2+14x \\
\boxed{-8x^2+12x}
\end{array}
$$

Here $-8x^2+2x=\boxed{-4x}$ and $-4x(2x-3)=\boxed{-8x^2+12x}$.

$$
\begin{array}{r}
2x^2\ -4x\ \boxed{+1} \\
2x-3\overline{)\,4x^3-14x^2+14x-3} \\
\underline{4x^3-6x^2} \\
-8x^2+14x \\
\underline{-8x^2+12x} \\
2x-3 \\
\boxed{2x-3} \\
\hline
0
\end{array}
$$

Here $2x+2x=\boxed{1}$ and $1(2x-3)=\boxed{2x-3}$.

Check:

$$(2x-3)(2x^2-4x+1)=4x^3-8x^2+2x-6x^2+12x-3$$
$$=4x^3-14x^2+14x-3$$

Do at least two of the following questions.

Divide each by long division.

1. $(x^2-5x+6)\div(x-2)$

2. $\dfrac{2x^2-5x-3}{x-3}$

3. $(x^3-4x^2+7x-6)\div(-x+2)$

4. $\dfrac{6x^3-11x^2+14x-5}{2x-1}$

For solutions to **Extensions**, turn to **Appendix A, Topic 3**.

To check that $(4x^3-14x^2+14x-3)\div(2x-3)=2x^2-4x+1$, either one of these methods can be used.

Method 1:

$$\begin{array}{l} 2x^2-4x+1 \\ \times \quad 2x-3 \\ \hline 4x^3-8x^2+2x \\ \quad -6x^2+12x-3 \\ \hline 4x^3-14x^2+14x-3 \end{array}$$

Method 2:

The expression $(2x-3)(2x^2-4x+1)$ can be calculated as follows:

$$2x\times(2x^2-4x+1)=4x^3-8x^2+2x$$
$$-3\times(2x^2-4x+1)=-6x^2+12x-3$$

After adding the resulting polynomials and collecting like terms, you get $4x^3-14x^2+14x-3$.

Topic 4 Factoring Polynomials

Introduction

Open and close, on and off, button and unbutton; within each of these pairs one is the opposite of the other. In fact, one undoes the other. Add and subtract and multiply and divide are examples of mathematical opposites. One operation undoes the other.

Expansion of polynomials is an operation of the multiplication and simplification of polynomials. The opposite of this operation is called **factoring**. Factoring undoes expansion or multiplication.

In this unit you will be exposed to several ways in which factoring can be done. Will polynomial tiles or binomial grids be the easiest method of factoring for you? Three methods of factoring are presented in this unit. Read on to find the method most suitable for you.

If you have access to a videocassette player, you may view the video titled *Freddie Factor*. This program will introduce you to the use of polynomial tiles for factoring.

What Lies Ahead

Throughout the topic you will learn to

1. factor polynomials with a common factor
2. factor trinomials of the form ax^2+bx+c, where $a=1$
3. recognize and factor perfect square trinomials
4. recognize and factor a difference of squares
5. factor polynomials which require a combination of methods in order to be fully factored

Now that you know what to expect, turn the page to begin your study of factoring polynomials.

Exploring Topic 4

Activity 1

Factor polynomials with a common factor.

Do you remember what a factor is? A factor is a number that divides evenly into another given number. For example, the factors of 15 are 1, 3, 5, and 15. Any algebraic term also has factors. For example, in $5x$ the factors are 1, 5, and x.

A polynomial may have a common factor in every term. You can remove this common factor and write the polynomial as a product of two factors. This is called **factoring**.

Example 1

Factor $2x^2 + 4y$.

Solution:

You should be able to see that 2 is a common factor in both terms.

Thus, you can write $2x^2$ as $2(x^2)$ and $4y$ as $2(2y)$.

Therefore, $2x^2 + 4y = 2(x^2 + 2y)$.

Check by multiplying.

$$2(x^2 + 2y) = 2(x^2) + 2(2y)$$
$$= 2x^2 + 4y$$

Multiplication undoes factoring. You can check your work in factoring by multiplying the factors you obtain.

You will learn to factor completely in this topic. There are many instances in which you can factor a polynomial, but the factoring is not complete. This means you can remove even more common factors out of the polynomial.

Look at the next example.

The G.C.F. or greatest common factor of 8 and 6 is 2 because the factors of 8 are 1, 2, 4, and 8 while the factors of 6 are 1, 2, 3, and 6.

Example 2

Factor $8x^2 + 16xy$ completely.

Solution:

You can factor 2 out of the polynomial to get $2(4x^2 + 8xy)$.

Since you can still factor $4x^2 + 8xy$, the original polynomial is not yet factored completely. Factor out another 2.

$$2(4x^2 + 8xy) = (2)(2)(2x^2 + 4xy)$$

Again you can factor $(2x^2 + 4xy)$ using 2 as another common factor.

Therefore,

$$(2)(2)(2x^2 + 4xy) = (2)(2)(2)(x^2 + 2xy).$$

Can you still factor $(x^2 + 2xy)$? Yes, you can since x is a factor of x^2 and $2xy$.

Therefore,

$$(2)(2)(2)(x^2 + 2xy) = (2)(2)(2)(x)(x + 2y).$$

Since $x + 2y$ has no further common factor, the final factored form is as follows:

$$8x^2 + 16xy = 8x(x + 2y)$$

Check:

$$8x(x + 2y) = 8x(x) + 8x(2y)$$
$$= 8x^2 + 16xy$$

This is the polynomial of the original form. Therefore, $8x^2 + 16xy$ becomes $8x(x + 2y)$ when completely factored.

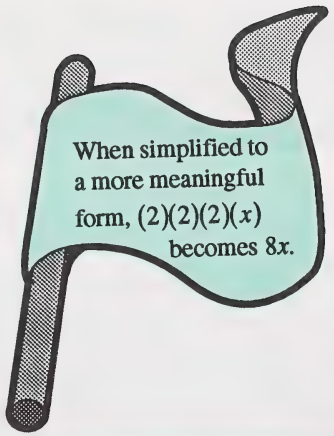
Greatest Common Factor is the term used to describe the factor which, when it is divided out from an expression, leaves an expression that cannot be factored further.

Factor completely has the same meaning as factoring out the greatest common factor.

Try the following questions.

Factor each polynomial completely and check by expanding.

1. $3x + 9$
2. $8y - 6$
3. $x^2 + xy$
4. $ax^2 + ay$



Always try to factor out the greatest common factor in one step where possible.

Note that $8x^2 + 16xy$ can be factored completely in one step. You can divide both terms evenly by $8x$ and get $\frac{8x^2}{8x} + \frac{16xy}{8x} = x + 2y$. From this you can see that the two factors of $8x^2 + 16xy$ are $8x$ and $x + 2y$.

5. $21x^2+14x+7$

6. $4x^3y+8x^2y^2-4xy^3$

7. $6x^4y^4+9x^3y^3-3x^2y^2$

8. The area of a rectangle is $(12x^2-36x)$ square units. The length of one side is $4x$ units. Using factoring, determine the length of the other side.

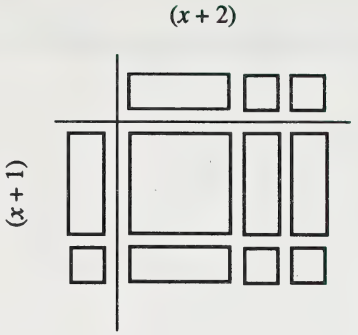
For solutions to **Activity 1**, turn to **Appendix A, Topic 4**.

Activity 2

Factor trinomials of the form ax^2+bx+c, where $a=1$.

You should remember multiplying a binomial by a binomial using polynomial tiles. Study the following for a quick review.

Multiply $(x+1)(x+2)$ using polynomial tiles.



Therefore, $(x+1)(x+2)=x^2+3x+2$.

Since the factoring of polynomials undoes multiplication, you can start with the products and work backwards to the factors. Actually this is quite simple since all you have to do is arrange the tiles related to the trinomial in a **rectangle**. Once you have made the rectangle, count the different tiles, put them together as a sum or difference, and read the factors at the side and at the top.

Review the **polynomial tiles** and what they represent.

- The flat represents an x^2.
- The strip represents an x.
- The unit represents a unit or 1.

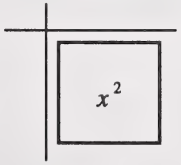
Example 3

Factor x^2+5x+6 using polynomial tiles.

Solution:

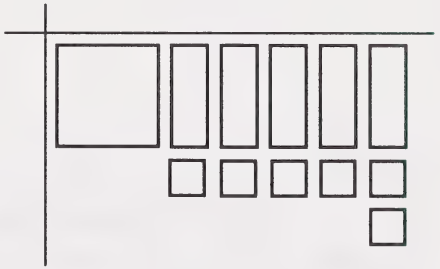
Work through these building steps, but if you immediately see the way the rectangle should be made, just move on to that final step.

Since there is only one x^2 term, you can have the x^2 tile in only one place.



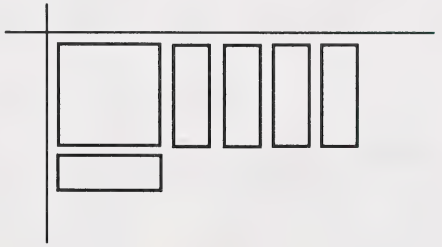
Now you have five x tiles to place in such a way that a rectangle is formed. There are a few possibilities. One possibility is having all five tiles placed straight across from the x^2 tile. This is shown as follows.

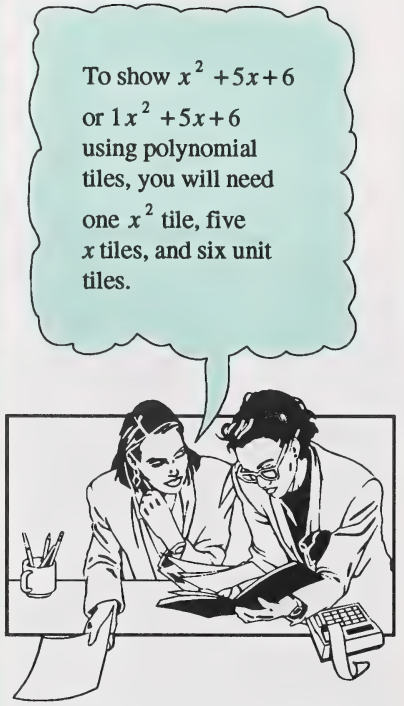
If you choose this arrangement, you now have to place six unit tiles in such a way that the result is a rectangle. You cannot place them in the following way since there is no rectangle after all placements are done.



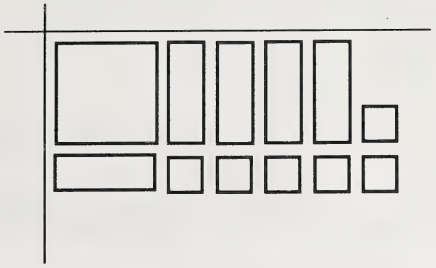
Therefore, the previous arrangement cannot be used.

Try putting the x tiles into this pattern.



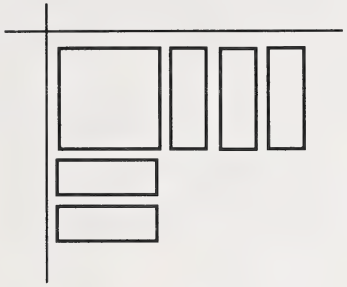


When placing the unit tiles, you get an arrangement like this.



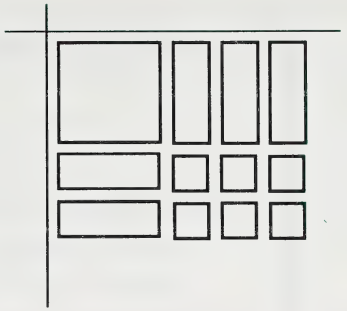
You have two unit tiles left over and you still do not have a rectangle.

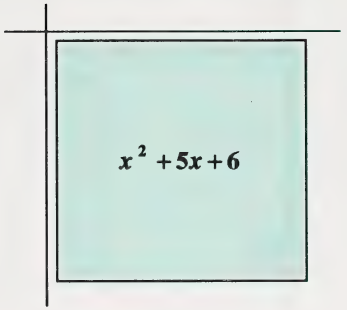
Another starting possibility is this.



This is a good choice since 3 and 2 are factors of 6 which add up to 5.

Now put the unit tiles in place to finally get a rectangle.



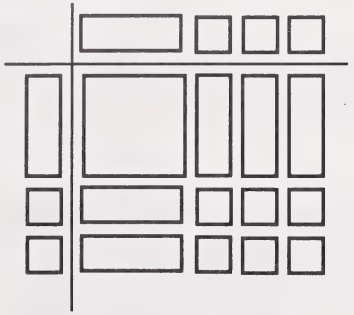


Do you think this is the answer? To check the work, count the tiles to see if there are enough of each shape.

Do you have one x^2 tile, five x tiles, and six unit tiles? Yes.
Do you have a rectangle? Yes.

All required conditions are met so you can now proceed to find the actual factors.

Draw in the factors as shown. The shapes along the side and along the top of this representation are now counted to determine the factors.



Therefore, the factors are $(x+2)$ and $(x+3)$.

Check:

$$(x+2)(x+3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Since the product is the same as the original polynomial, the factors are correct.

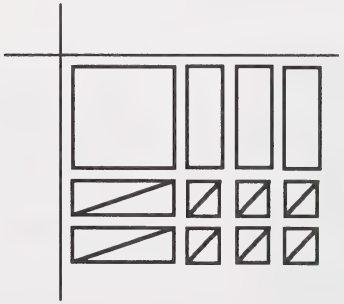
Study another example.

Example 4

Factor $x^2 + x - 6$.

Solution:

Is it possible to make a rectangle with exactly one x^2 tile, one x tile, and six negative unit tiles? The answer is no since no arrangement of only these tiles will result in a rectangle.

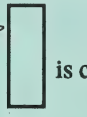


How did this diagram come from $x^2 + x - 6$?

You know that you need one flat for x^2. Since $-2x + 3x = x$, you can use **two negative strips** and **three positive strips** to make x. You also need six negative units for -6.

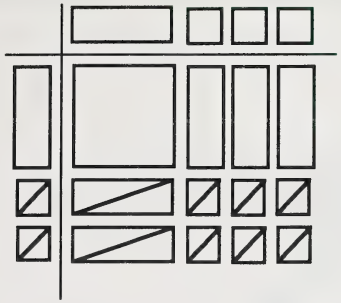
Remember that each tile has a unique name.

A is called a flat,

a  is called a strip, and

a is called a unit.

Draw in the factors.



Therefore, the factors are $(x-2)$ and $(x+3)$.

Check: $(x-2)(x+3) = x^2+3x-2x-6$

$= x^2+x-6$

Do at least four of the following questions. Do the rest if you want more practice.

Factor the following using polynomial tiles.

1. $x^2+7x+12$
2. x^2-x-2
3. x^2-3x+2
4. x^2+x-6
5. x^2-3x-4
6. x^2+4x-5

For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.

Using polynomial tiles is a good hands-on method to help you understand the process of factoring. The binomial grid provides a similar method for factoring trinomials using a pencil and paper.

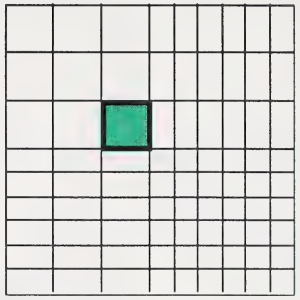
Example 5

Factor $x^2+8x+15$ using a binomial grid.

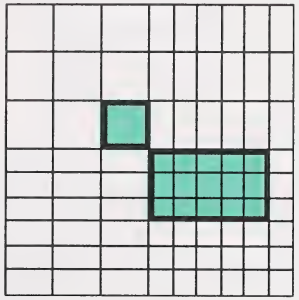
Solution:

Try to make a rectangle which includes all of the terms in the trinomial.

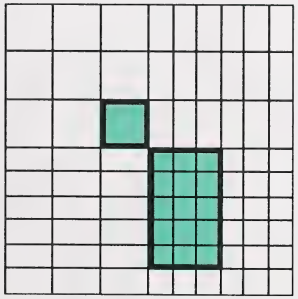
Start with the first term which is x^2.



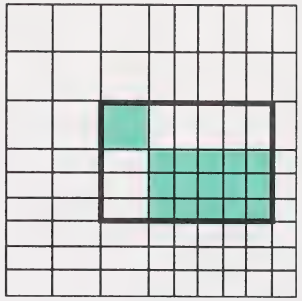
Now draw in the term 15 starting at the lower right corner of the x^2 flat. There are only two possible arrangements of the unit shapes. These arrangements are 5×3 and 3×5. On the following grid a 5×3 arrangement is used.



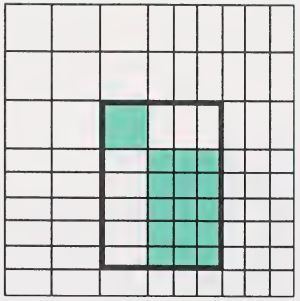
A 3×5 arrangement would look like this.



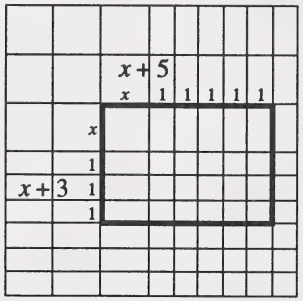
Now complete the sides of the rectangle.

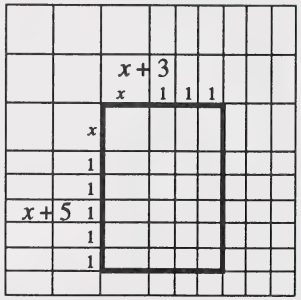


or



Count the number of x terms and unit terms on each side.





Therefore, the factors of $x^2+8x+15$ are $(x+3)$ and $(x+5)$ or $(x+5)$ and $(x+3)$.

Check: $(x+3)(x+5)=x^2+5x+3x+15$

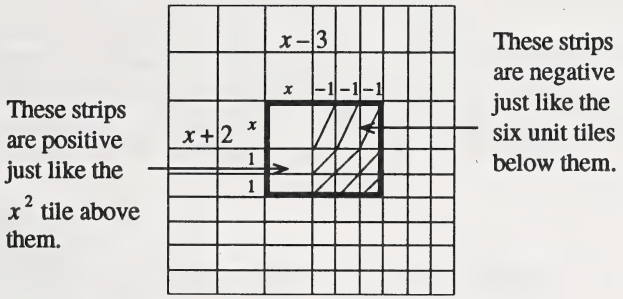
$=x^2+8x+15$

Example 6

Factor $x^2 - x - 6$ using a binomial grid.

Solution:

Start a rectangle and place x^2 in one corner and -6 in the other corner. Finish the rectangle and count the number of x terms and unit terms on each side.


x − 3
x
−1 −1 −1
x + 2
x
1
1
These strips are positive just like the x² tile above them.
These strips are negative just like the six unit tiles below them.

Therefore, the factors of $x^2 - x - 6$ are $(x+2)$ and $(x-3)$.

Check:

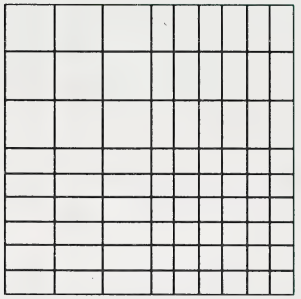
$$(x+2)(x-3) = x^2 - 3x + 2x - 6$$
$$= x^2 - x - 6$$

Try to factor some similar polynomials on your own now.

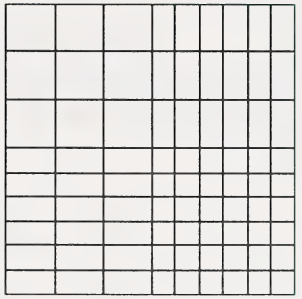
Do at least two of the following questions.

Factor using a binomial grid.

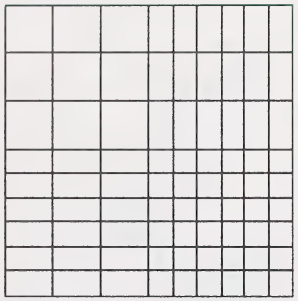
7. $x^2 + 2x + 1$



8. $x^2 + 5x + 4$



9. $x^2 - 2x - 3$



For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.

If you watched closely, you may have noticed a pattern and a relationship between the factors and the product. This is a trinomial. If you missed seeing the pattern, the following step-by-step explanation will try to identify the pattern so that you can use it when factoring any trinomial.

Expand $(x+2)(x+3)$.

You get the following:

$$(x+2)(x+3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Now look at these circled numbers.

$$(x+\textcircled{2})(x+\textcircled{3}) = x^2 + 5x + \textcircled{6}$$

$$\textcircled{2} \times \textcircled{3} = \textcircled{6}$$

Next look at these circled numbers.

$$(x+\textcircled{2})(x+\textcircled{3}) = x^2 + \textcircled{5}x + 6$$

$$\textcircled{2} + \textcircled{3} = \textcircled{5}$$

The **numerical coefficient of the middle term** of the trinomial product is the sum of the last terms of the binomial factors.

The **last term of the trinomial** product is the product of the last terms of the two binomial factors.

Try to predict the numerical coefficient of the middle term and the last term in the trinomial product resulting from the multiplication of these binomials.

$$(x+2)(x+4) = x^2 + \_\_\_\_ x + \_\_\_\_$$

The numerical coefficient of the middle term is $2 + 4 = 6$.
The last term is $2 \times 4 = 8$.
Therefore, the expansion should be $(x+2)(x+4) = x^2 + 6x + 8$.

Check: $(x+2)(x+4) = x^2 + 4x + 2x + 8$
$$= x^2 + 6x + 8$$

You should notice that both trinomial products are exactly the same.

Do at least questions 10 to 13. Do the others if you want more practice.

Predict and fill in the blank values. Include the correct sign with your prediction. (Note in question 11 that $x^2+0x-16$ is the same as x^2-16.)

10. $(x+7)(x+6)=x^2$ ____ x ____

11. $(x+4)(x-4)=x^2$ ____ x ____

12. $(x-2)(x-3)=x^2$ ____ x ____

13. $(x-8)(x+6)=x^2$ ____ x ____

14. $(y-3)(y+5)=y^2$ ____ y ____

15. $(z-9)(z-9)=z^2$ ____ z ____

For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.

This pattern should be clear in your mind before you start this next section. Now you will learn to perform the opposite operation.

You are going to look for two numbers such that when added they give the numerical coefficient of the second term and when multiplied they give the third term of the trinomial product.

Example 7

Factor x^2+4x+3.

Solution:

You know that the binomial factors will look like $(x+\_\_\_\_)(x+\_\_\_\_)$. You need to look for two numbers which when added give positive 4 and when multiplied result in positive 3.

The only numbers that multiply to give positive 3 and add to give positive 4 are 1 and 3.

$1\times 3=3$
$1+3=4$

Therefore, the factors are $(x+1)$ and $(x+3)$.

Check:

$$(x+1)(x+3)=x^2+3x+1x+3$$
$$=x^2+4x+3$$

The factors $(x+1)(x+3)$ can also be arranged as $(x+3)(x+1)$.

Example 8

Factor x^2+x-2.

Solution:

Find two numbers that add to give 1 and multiply to give 2.

The factors of -2 must be -2 and 1 or 2 and -1. Select the correct pair which will give the sum of 1. Since $2+(-1)=1$, the two binomial factors are $(x+2)$ and $(x-1)$.

Check: $(x+2)(x-1)=x^2-x+2x-2$

$=x^2+x-2$

Example 9

Factor x^2-5x+6.

Solution:

Find two numbers that add to give -5 and multiply to give $+6$.

The possible factors of 6 are as follows:

- 6 and 1 because $6\times1=6$
- -6 and -1 because $-6\times(-1)=6$
- 3 and 2 because $3\times2=6$
- -3 and -2 because $-3\times(-2)=6$

Select the factors that add to give -5. They are -3 and -2.

Therefore, the two factors are $(x-3)$ and $(x-2)$.

Check:

$(x-3)(x-2)=x^2-2x-3x+6$

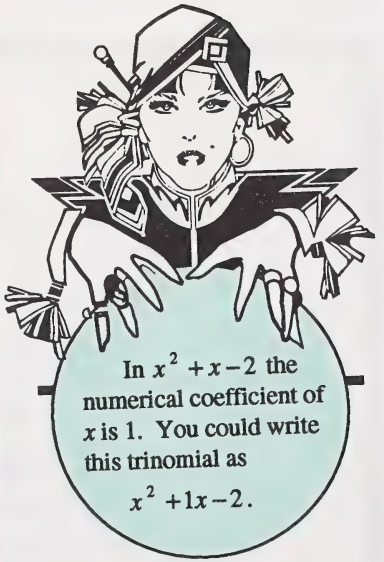
$=x^2-5x+6$

Do at least seven of the following questions. Do the rest if you need more practice.

Factor using algebraic methods only.

16. $x^2+11x+10$
17. x^2+6x+8
18. $x^2+7x+12$
19. $x^2-2x-15$
20. $x^2-9x+18$
21. x^2-4x+4
22. x^2-5x-6
23. y^2-3y-4
24. $z^2-9z+20$
25. a^2+2a-3

For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.



In x^2+x-2 the numerical coefficient of x is 1. You could write this trinomial as x^2+1x-2.

When you give the factors of x^2-5x+6, you can write them as either $(x-3)(x-2)$ or $(x-2)(x-3)$ since you can multiply in any order you choose.

If a polynomial appears to be too complicated to factor by the method given previously, it could be that there is a common factor that must first be removed from the polynomial before the two binomial factors can be determined.

Example 10

Factor $3x^2+33x+30$.

Solution:

Notice that 3 is a common factor of all three terms. It is removed to give

$3x^2+33x+30=3\left(x^2+11x+10\right)$.

Now you can factor $x^2+11x+10$ to $(x+10)(x+1)$.

Therefore,

$3x^2+33x+30=3(x+10)(x+1)$ when written in complete factored form.

Example 11

Factor $2x^2y-10xy+12y$.

Solution:

You must first remove the highest common factor from all three terms.

Change $2x^2y-10xy+12y$ to $2y\left(x^2-5x+6\right)$.

Now factor the trinomial $\left(x^2-5x+6\right)$ to get $(x-3)(x-2)$.

Therefore,

$2x^2y-10xy+12y=2y(x-3)(x-2)$ when written in complete factored form.

Do at least seven of the following questions. Do the rest if you need more practice.

Factor each trinomial completely.

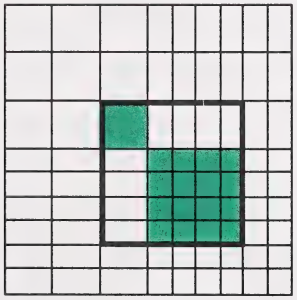
26. $7x^2-21x+14$
27. $3b^2+18b-48$
28. $2y^2-14y+20$
29. $4x^2+16x+12$
30. $5a^2-20a+20$
31. $2x^2+6x-56$
32. $5x^2+10x-75$
33. $2b^2-16b-18$
34. x^3-6x^2+8x
35. $3z^2-15z-42$

For solutions to **Activity 2**, turn to **Appendix A, Topic 4**.

Activity 3

Recognize and factor perfect square trinomials.

Look at the following binomial grid which shows $x^2+8x+16$.



Notice that the trinomial is represented by a square rather than a rectangle. Since the sides of the square are the same length, the factors are also equal.

$$x^2+8x+16=(x+4)(x+4)$$
$$=(x+4)^2$$

How can you tell if a trinomial is a perfect square trinomial?

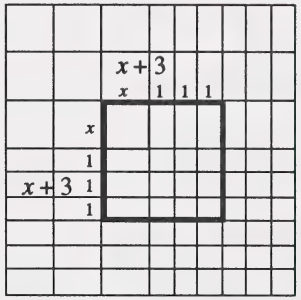
A trinomial is a perfect square trinomial if the square root of the third term is a perfect square and the numerical coefficient of the second term is twice the square root of the third term.

Example 12

Is x^2+6x+9 a perfect trinomial?

Solution:

Since $\pm\sqrt{9}=\pm 3$ and $3\times 2=6$, this is a perfect square trinomial. You can represent the trinomial on a binomial grid.



This forms a square with sides of $x+3$. Therefore, the factors are $(x+3)$ and $(x+3)$. This can be written as $x^2+6x+9=(x+3)(x+3)=(x+3)^2$.

Always study the key items before proceeding.

A perfect square trinomial can be factored into a single binomial that is squared.

Example 13

Is $x^2+15x+25$ a perfect square trinomial?

Solution:

$\pm\sqrt{25}=\pm 5$ but $5\times 2\neq 15$

Therefore, $x^2+15x+25$ is not a perfect square trinomial.

Example 14

Is x^2-6x+9 a perfect square trinomial?

Solution:

$\pm\sqrt{9}=\pm 3$ and $(-3)\times 2=-6$

Therefore, x^2-6x+9 is a perfect square trinomial and $x^2-6x+9=(x-3)(x-3)$

$=(x-3)^2$.

Do at least seven of the following questions.

Determine whether or not each of the following is a perfect square trinomial and factor any that can be factored.

1. $x^2-8x+16$
2. $x^2+10x+25$
3. y^2+4y+4
4. x^2-2x+1
5. $x^2+5x+\frac{25}{4}$
6. $2x^2-12x+18$
7. $(x+2)^2+4(x+2)+4$
8. $3x^2-18x+27$
9. $x^2+10x+16$
10. x^2+25

For solutions to **Activity 3**, turn to **Appendix A, Topic 4**.

For question 5 you will find that $\pm\sqrt{\frac{25}{4}}=\pm\frac{5}{2}$ since $\frac{5}{2}\times\frac{5}{2}=\frac{25}{4}$ and $\frac{-5}{2}\times\frac{-5}{2}=\frac{25}{4}$.

Using $\frac{5}{2}$ to find the coefficient of the middle term, you get this:

$$\frac{5}{2}\times 2=\frac{5}{2}\times\frac{2}{1}$$
$$=\frac{10}{2}$$
$$=5$$

Thus, the middle term is $5x$.

From $\sqrt{9}=\pm 3$, the root of -3 gives a middle term coefficient of $(-3)\times 2=-6$ while the other root of $+3$ would give a middle term coefficient of $(+3)\times 2=6$.

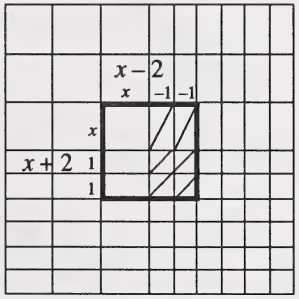
Activity 4

Recognize and factor a difference of squares.

Expand the binomials $(x+2)$ and $(x-2)$.

$$(x+2)(x-2) = x^2 - 2x + 2x - 4$$
$$= x^2 - 4$$

If you represent $x^2 - 4$ on the binomial grid, you get the following.



A polynomial that is expressed as the difference of two squares is easy to recognize. This polynomial must consist of two terms or a binomial with a subtraction sign between them. Examples of polynomials that are a difference of two squares are $x^2 - 25$, $x^4 - 9$, $4y^2 - 36$, and $100x^2 - 25y^2$.

Try making up your own examples if you like.

Within the requirements of this topic you will be limited to working with the difference of two squares where both terms are perfect squares. You will be working with binomials like $x^2 - 16$ where both x^2 and 16 are perfect squares.

Binomials like $x^2 - 8$ are still the difference of two squares, but because 8 is not a perfect square, it is more complicated to factor it.

How can you tell if a binomial is a difference of squares?

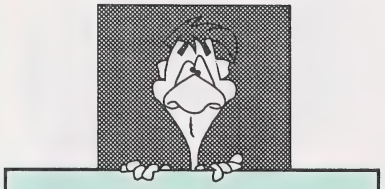
Example 15

Is $x^2 - 25$ a difference of squares?

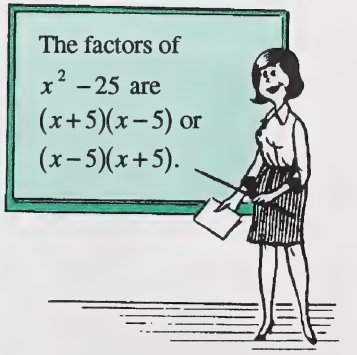
Solution:

Yes, $x^2 - 25$ is a difference of squares because $\pm\sqrt{x^2} = \pm x$ and $\pm\sqrt{25} = \pm 5$ and the terms are separated by a subtraction sign.

You can factor $x^2 - 25$ into the factors of $(x+5)$ and $(x-5)$.



Note that $x^2 - 8$ is a difference of squares because it can be factored to become $\left(x+\sqrt{8}\right)\left(x-\sqrt{8}\right)$ since $\sqrt{8} \times \sqrt{8} = 8$.



Example 16

Is $3y^2+16$ a difference of squares?

Solution:

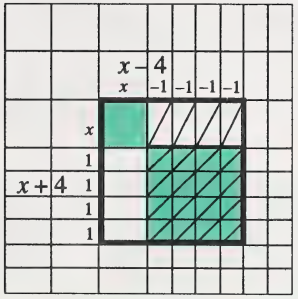
No, $3y^2+16$ is not a difference of squares. There is an addition sign between the two terms and $3y^2$ is not a perfect square.

Example 17

Is x^2-16 a difference of squares?

Solution:

Yes, x^2-16 is a difference of squares since $\pm\sqrt{x^2}=\pm x$ and $\pm\sqrt{16}=\pm 4$ and there is a subtraction sign between x^2 and 16. The factors of x^2-16 are $(x+4)$ and $(x-4)$. You can represent x^2-16 on a binomial grid as shown.



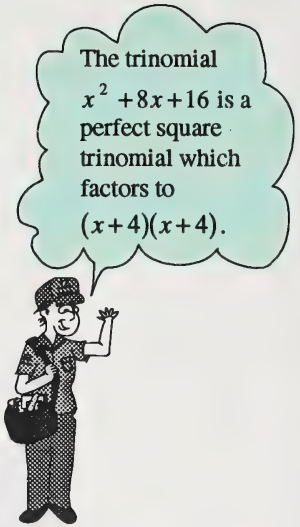
This representation is a square as was the perfect square trinomial. The only difference is that the x tiles cancel each other since $4x-4x=0$.

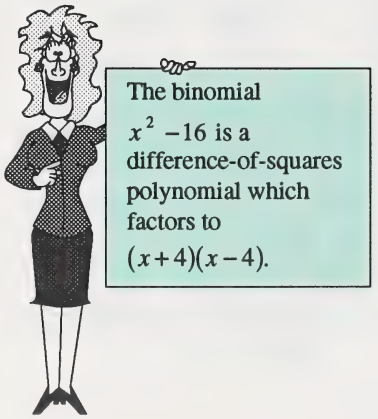
Do at least six of the following questions.

State whether or not each of the following is a difference of squares and factor it if it is.

1. x^2-25
2. y^2+16
3. x^2-4x-4
4. x^2-9
5. $x^2-\frac{9}{4}$
6. x^2-144
7. x^2-y^2
8. $16x^4-9y^8$
9. $(x-2)^2-(y+2)^2$

For solutions to **Activity 4**, turn to **Appendix A, Topic 4**.





Activity 5

Factor polynomials which require a combination of methods in order to be fully factored.

Here is the order of steps you should use when factoring polynomials.

Step 1: Factor out the highest common factor if possible.

Step 2: Determine the type of polynomial that is to be factored.

- Is it a perfect square trinomial such as x^2+6x+9?
- Is it a difference-of-squares binomial such as x^2-y^2?
- Is it a factorable trinomial such as x^2-3x+2?

Step 3: Factor using polynomial tiles, a binomial grid, or an algebraic method, bearing in mind which type of polynomial you are factoring.

Example 18

Factor $5x^2y-5xy-30y$ completely.

Solution:

Step 1: Factor out the highest common factor which is $5y$.

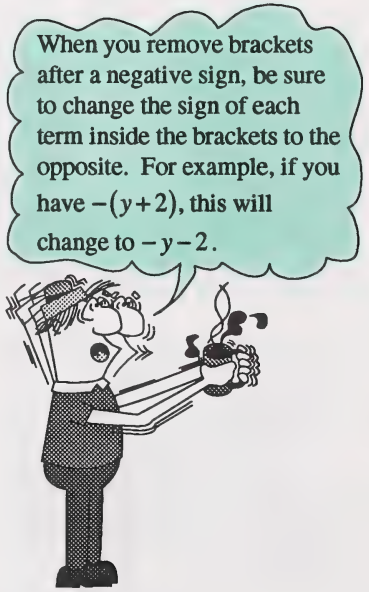
$$5x^2y-5xy-30y=5y\left(x^2-x-6\right)$$

Step 2: The trinomial is not a perfect square nor a difference of squares which would be a binomial, so you must try to factor as you did in Activity 2.

Because $+2+(-3)=-1$ and $+2\times(-3)=-6$, it follows that $\left(x^2-x-6\right)=(x+2)(x-3)$.

Therefore,

$5x^2y-5xy-30y=5y(x+2)(x-3)$ when factored completely.


When you remove brackets after a negative sign, be sure to change the sign of each term inside the brackets to the opposite. For example, if you have $-(y+2)$, this will change to $-y-2$.

Example 19

Factor $x^3 - 4x^2 + 4x$ completely.

Solution:

Step 1: Factor out the highest common factor which is x.

$$x^3 - 4x^2 + 4x = x\left(x^2 - 4x + 4\right)$$

Step 2: Since $\left(x^2 - 4x + 4\right)$ is a perfect square trinomial, it becomes as follows when factored.

$$x^2 - 4x + 4 = (x-2)(x-2)$$
$$= (x-2)^2$$

Therefore, $x^3 - 4x^2 + 4x = x(x-2)^2$ when factored completely.

Example 20

Factor $x^4 - a^2x^2$ completely.

Solution:

Step 1: Factor out the highest common factor which is x^2.

$$x^4 - a^2x^2 = x^2\left(x^2 - a^2\right)$$

Step 2: Since $\left(x^2 - a^2\right)$ is a difference-of-squares binomial, it becomes $\left(x^2 - a^2\right) = (x-a)(x+a)$ when factored.

Therefore, $x^4 - a^2x^2 = x^2(x-a)(x+a)$ when factored completely.

Do at least four of the following questions.

Factor completely.

1. $8x^2 - 4$
2. $2x^2y - 16$
3. $3x^2y^2 - 9xy^2 + 6y^2$
4. $2y(x+3) - 4(x+3)$
5. $3a\left(x^2 - 2\right) - 9a$
6. $x^2(2x-1) - 3x(2x-1) - 10(2x-1)$

For solutions to **Activity 5**, turn to **Appendix A, Topic 4**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Are you still having problems factoring? Fear not, for here is a student-developed technique that provides a structured and systematic method for factoring trinomials and special-case binomials. This method is called the L method. You will be led step by step through the method using various examples.

Example 21

Factor $x^2+2x-15$.

Solution:

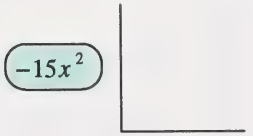
Draw a big L to begin.

Step 1: Multiply the first and last terms in the trinomial.

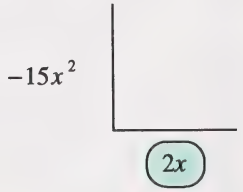
first term → x^2 $+2x$ -15 ← last term

$x^2 \times (-15) = -15x^2$

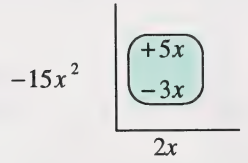
Write this term left of the L as shown here.



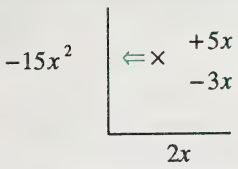
Step 2: Write the **middle** term which is $2x$ below the L as shown here.



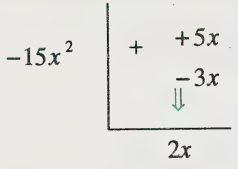
Step 3: Find two monomials that multiply to give the term left of L $\left(-15x^2\right)$ and that also add to give the term below L $(2x)$. Place them in the L as shown here.



The idea looks like this.



The monomials inside the L **multiply** to give the term left of the L.



The monomials inside the L **add** to give the term at the bottom of the L.

Step 4: Write the first term of the original polynomial followed by the two terms in the middle of the L, followed by the last term of the original polynomial and you get

$x^2+5x-3x-15$.

Step 5: Group the terms and factor.

$$\underbrace{x^2+5x}\ \ \underbrace{-3x-15}$$

$$x(x+5)-3(x+5)=(x-3)(x+5)$$

Check your factoring by expansion.

$$(x-3)(x+5)=x^2+5x-3x-15$$
$$=x^2+2x-15$$

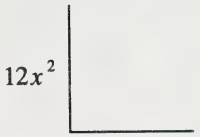
Therefore, the factors of $x^2+2x-15$ are $(x-3)(x+5)$.

Example 22

Factor $x^2+7x+12$ using the L method.

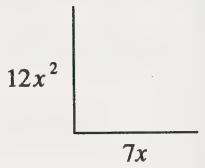
Solution:

Step 1:



$12\times x^2=12x^2$

Step 2:



middle term = $7x$

Step 3: There are three possibilities to get $12x^2$.

$(12x)(1x)$
$(6x)(2x)$
$(4x)(3x)$

Go through all three possibilities.

$12x^2$	$12x$
	$1x$
	$7x$

$12x^2$	$6x$
	$2x$
	$7x$

These two do not work.

Remember the middle two terms must multiply to give $12x^2$ and add to give $7x$.

$12x^2$	$4x$
	$3x$
	$7x$

This one does work.

Step 4: $x^2+4x+3x+12$

Step 5: $\left(x^2+4x\right)+(3x+12)$

$=x(x+4)+3(x+4)$

$=(x+3)(x+4)$

Check:

$$(x+3)(x+4)=x^2+4x+3x+12$$
$$=x^2+7x+12$$

Therefore, the factors are $(x+3)$ and $(x+4)$.

Do at least three of the following questions.

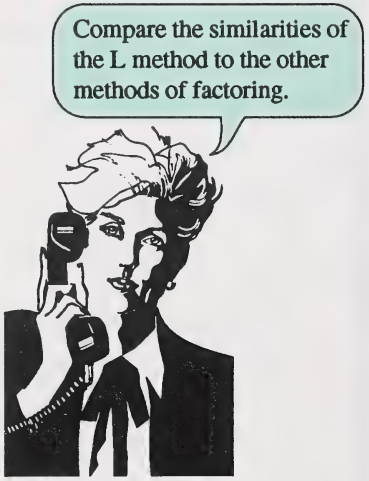
Factor the following using the L method.

1. $x^2+2x-15$
2. $x^2+3x-10$
3. $x^2-8x+12$
4. $x^2+14x+49$
5. x^2-y^2

For solutions to **Extra Help**, turn to **Appendix A, Topic 4**.

If you have access to an Apple II plus or higher Apple II series computer, you may use the Apple II diskette entitled *Algebra Drill & Practice II* [1] by Conduit. Go to program six on disk 2 and work through as much as you find necessary.

[1] *Algebra Drill & Practice II* is a title of Conduit.



Extensions

Factoring has its uses especially in mathematics and other fields strongly dependent on mathematics such as engineering and science.

You can simplify complicated expressions using factoring.

Example 23

Simplify $\frac{x^2-4}{x+2}$.

Solution:

If you have common factors in the numerator and denominator, you can divide out the common factor.

Since $x^2-4=(x+2)(x-2)$, you can write $\frac{x^2-4}{x+2}$ as $\frac{(x+2)(x-2)}{(x+2)}$.

Now divide out the common factor $(x+2)$ to get the following:

$$\frac{x^2-4}{x+2}=\frac{\left(\overset{1}{\cancel{x+2}}\right)(x-2)}{\left(\underset{1}{\cancel{x+2}}\right)}$$

$$=x-2$$

Therefore, $\frac{x^2-4}{x+2}=x-2$.

You can add or subtract complex fractions using factoring to help you find the lowest common denominator.

Example 24

Add $\frac{1}{x^2-3x}+\frac{1}{x-3}$.

Solution:

Factor the denominators to help find the lowest common denominator.

$$x^2-3x=x(x-3)$$

The denominator $x-3$ is not factorable.

Now you can rewrite $\frac{1}{x-3}$ as a fraction with $x(x-3)$ as the denominator. In order to have $x(x-3)$ as the denominator, you need to multiply $\frac{1}{x-3}$ by $\frac{x}{x}$.

$$\frac{1}{x-3}\times\frac{x}{x}=\frac{x}{x(x-3)}$$

Add the fractions.

$$\frac{1}{x^2-3x}+\frac{1}{x-3}=\frac{1}{x(x-3)}+\frac{x}{x(x-3)}$$
$$=\frac{1+x}{x(x-3)}$$
$$=\frac{1+x}{x^2-3x}$$
$$=\frac{x+1}{x^2-3x}$$

There are some interesting number tricks that involve factoring.

Example 25

Multiply 49×51 mentally.

Solution:

Notice that $49=50-1$ and $51=50+1$. Therefore, you can write 49×51 as $(50-1)(50+1)$.

This is a difference of squares where $(50-1)(50+1)=50^2-1^2$.

Now it is easy to solve 49×50 using the following:

$$(50-1)(50+1)=50^2-1^2$$
$$=2500-1$$
$$=2499$$

There are polynomials that can be factored but must first have the terms grouped so that the method of factoring that should be used is clear.

Example 26

Factor $xy+2x+3y+6$.

Solution:

Group and factor each group.

$$(xy+2x)+(3y+6)=x(y+2)+3(y+2)$$

Now factor out the binomial $(y+2)$.

$$x(y+2)+3(y+2)=(y+2)(x+3)$$

Therefore, $xy+2x+3y+6=(y+2)(x+3)$.

Check:

$$(y+2)(x+3)=yx+3y+2x+6$$
$$=xy+2x+3y+6$$

This is the same as what was done in the last step of the L method in the **Extra Help** section.

Example 27

Factor $2xy+2x^2-3x-3y$.

Solution:

You first have to rearrange the terms to make grouping possible.

$$2xy+2x^2-3x-3y=2x^2-3x+2xy-3y$$

Now group the first two terms and the last two terms to get $(2x^2-3x)+(2xy-3y)$.

Now factor each group.

$$(2x^2-3x)+(2xy-3y)=x(2x-3)+y(2x-3)$$
$$=(2x-3)(x+y)$$

Therefore, the factored form for $2xy+2x^2-3x-3y$ is $(2x-3)(x+y)$.

Do at least three of the following questions.

1. Simplify each of the following.

a. $\dfrac{x^2-2x+1}{x-1}$ b. $\dfrac{x^2-4}{x^2+4x+4}$ c. $\dfrac{x^2-5x+6}{x^2+2x-15}$ d. $\dfrac{y^2-1}{y-1}$

2. Add or subtract and simplify completely.

a. $\dfrac{4}{xy}+\dfrac{5}{x}$ b. $\dfrac{2}{x^2-1}+\dfrac{3}{x^2-2x+1}$

c. $\dfrac{x^2-9}{x^2-5x+6}-\dfrac{3}{x-2}$ d. $\dfrac{2x}{4x^2+8x}-\dfrac{x}{2x+4}$

3. Mentally multiply each of the following.

a. 99×101 b. 499×501 c. 506×494

4. By grouping, factor each of the following completely.

a. $xy-5y+2x-10$ b. $3x^2-5xy+5y-3x$

For solutions to **Extensions**, turn to **Appendix A, Topic 4.**

Unit Summary

What You Have Learned

In this unit you have learned to

- identify the terminology of polynomials such as term, variable, monomial, and numerical coefficient
- evaluate polynomials given the values of the variables
- write polynomials which describe mathematical situations
- use the Product Law of exponents
- use the Power-of-a-Power Law of exponents
- use the Quotient Law of exponents
- use zero exponents and negative exponents
- add and subtract polynomials and arrange terms in alphabetical and descending order with respect to degree
- multiply polynomials by monomials
- multiply binomials by binomials
- divide polynomials by monomials
- factor polynomials with a common factor
- factor trinomials of the form $ax^2 + bx + c$, where $a = 1$.
- recognize and factor perfect square trinomials
- recognize and factor a difference of squares
- factor polynomials using a combination of methods

You are now ready to

complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Evaluating Polynomials

Topic 2 Power Laws for Exponents

Topic 3 Operations with Polynomials

Topic 4 Factoring Polynomials

Appendix B
Tiles and Grids

Polynomial Tiles

Polynomial Grids

Appendix A
Solutions

Review

1. a. $2p+3q=2(3)+3(5)$
$=6+15$
$=21$

b. $3r-3p=3(-2)-3(3)$
$=-6-9$
$=-15$

c. $2-2r=2-2(-2)$
$=2+4$
$=6$

d. $\frac{p+q}{r}=\frac{3+5}{-2}$
$=\frac{8}{-2}$
$=-4$

e. $3p-q=3(3)-(5)$
$=9-5$
$=4$

2. a. $(4^3)(2^2)=64\times4$
$=256$

b. $\left(\frac{1}{2}\right)^3(-2)^5=\frac{1}{8}\times(-32)$
$=-4$

c. $(3^2)(5^0)=9\times1$
$=9$

d. $4^{-1}=\frac{1}{4}$

e. $\left(\frac{1}{3}\right)^{-2}=\left(\frac{3}{1}\right)^2$
$=\frac{9}{1}$
$=9$

f. $5^3+5^2=5^{3-2}$
$=5^1$
$=5$

g. $4^3+8^2=64+64$
$=1$

h. $(-2)^4(3)^2+4^2=(16)(9)+16$
$=144+16$
$=9$

i. $\left(\frac{3}{4}\right)^{-2}=\left(\frac{4}{3}\right)^2$
$=\frac{4}{3}\times\frac{4}{3}$
$=\frac{16}{9}$
$=1\frac{7}{9}$

3. a. The greatest common factor of 9, 18, and 27 is 9

since $9 = \boxed{3\times3}$,
$18 = \boxed{3\times3}\times2$,
$27 = \boxed{3\times3}\times3$, and $3\times3 = 9$.

b. The greatest common factor of 18, 24, and 30 is 6

since $18 = \boxed{2\times3}\times3$,
$24 = \boxed{2\times3}\times2\times2$,
$30 = \boxed{2\times3}\times5$, and $2\times3 = 6$.

Exploring Topic 1

Activity 1

Identify the terminology of polynomials such as term, variable, monomial, and numerical coefficient.

1. The expressions in a, c, d, and f are terms, but b is not a term since $2x - y$ is the difference of two terms.

   Note that e is not a term since $ax^2 + b$ is the sum of two terms.

2. The variables for each are given as follows:

   a. x
   b. x, y
   c. x, y, z
   d. There are no variables in the term -3.
   e. a, b, x
   f. x, y

3. a. The terms are $3x^2y$, $-2xy^2$, $5y^3$, and -7.

   b. For $3x^2y$, the numerical coefficient is 3.
   For $-2xy^2$, the numerical coefficient is -2.
   For $5y^3$, the numerical coefficient is 5.
   For -7, the numerical coefficient is -7.

4. a. This is a trinomial since it has three terms.
   b. This is a binomial since it has two terms.
   c. This is a trinomial since it has three terms.
   d. This is a monomial since it has one term.
   e. This is a binomial since it has two terms.
   f. This is a trinomial since it has three terms.

Activity 2

Evaluate polynomials given the values of the variables.

1. $3x^2 - 4y = 3(4)^2 - 4(-2)$
$= 3(16) - 4(-2)$
$= 48 - (-8)$
$= 48 + 8$
$= 56$

2. $3y - x = 3(-2) - 4$
$= -6 - 4$
$= -10$

3. $y^4 - x^2 = (-2)^4 - (4)^2$
$= 16 - (16)$
$= 16 - 16$
$= 0$

4. $2y^2 - 3xy + 2 = 2(-2)^2 - 3(4)(-2) + 2$
$= 2(4) - 3(-8) + 2$
$= 8 + 24 + 2$
$= 34$

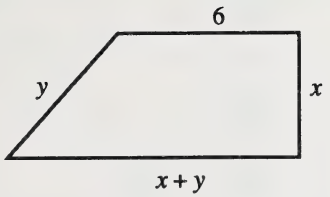
5. $4y^2 + 4x = 4(-2)^2 + 4(4)$
$= 4(4) + 4(4)$
$= 16 + 16$
$= 32$

6. $x^3y + x^2y^2 + xy^3 + y^4$
$= (4)^3(-2) + (4)^2(-2)^2 + (4)(-2)^3 + (-2)^4$
$= (64)(-2) + (16)(4) + (4)(-8) + 16$
$= -128 + 64 - 32 + 16$
$= -160 + 80$
$= -80$

Activity 3

Write polynomials which describe mathematical situations.

1.



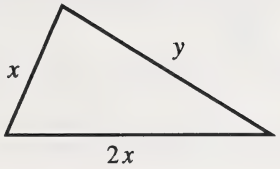
$P = y + 6 + x + (x + y)$

or

$P = y + 6 + x + x + y$

$P = 2x + 2y + 6$

2.

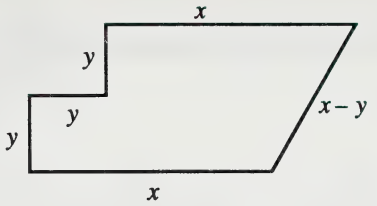


$P = x + y + 2x$

or

$P = 3x + y$

3.



$P = y + y + y + x + x + (x - y)$

or

$P = y + y + y + x + x + x - y$

$P = 2y + 3x$

Extra Help

1. a. The terms in $4x^2 - 5xy + 7y^2$ are $4x^2$, $-5xy$, and $7y^2$.

   b. The terms in $a - b^2 + 3ab - 5a^2b^2 + 7$ are a, $-b^2$, $3ab$, $-5a^2b^2$, and 7.

   c. The terms in $z^4 - 3z^2 - 9z + 12$ are z^4, $-3z^2$, $-9z$, and 12.

2. a. The polynomial $3a^2 - 5a + 6$ is a trinomial.

   b. The polynomial $\frac{-a^2}{4}$ is a monomial.

c. The polynomial $x^3 - 3x^2 + 2x - 4$ is a four-term polynomial.

d. The polynomial $m - 2n$ is a binomial.

e. The polynomial $4abc$ is a monomial.

f. The polynomial $a^2 - 2ab + b^2$ is a trinomial.

g. The polynomial $b^5 - 3a^2$ is a binomial.

h. The polynomial $x^2y^2 - 3x^2 + y^2 - 4xy$ is a four-term polynomial.

i. The polynomial $6r^2s^3t^4$ is a monomial.

j. The polynomial $4m^2 - 8mn + 2n^2$ is a trinomial.

3. a. $4x - 2y + z = 4(3) - 2(-1) + 2$
$= 12 + 2 + 2$
$= 16$

b. $x^2 - y^4 + z = (3)^2 - (-1)^4 + 2$
$= 9 - 1 + 2$
$= 11 - 1$
$= 10$

c. $x^3 - z^3 = (3)^3 - (2)^3$
$= 27 - 8$
$= 19$

d. $2xy + 3xz = 2(3)(-1) + 3(3)(2)$
$= 2(-3) + 3(6)$
$= -6 + 18$
$= 12$

Extensions

$A = P(1+r)^n$

$= 500.00\left(1 + 15\frac{1}{4}\%\right)^4$

$= 500.00(1 + 0.1525)^4$

$= 500.00(1.1525)^4$

$= 500.00(1.764\,264\,666)$

$= \$882.13$

To find the interest, you subtract the principal from the amount.

$\$882.13 - \$500.00 = \$382.13$

Therefore, the interest amounts to $382.13.

You can also use a calculator and follow these steps.

Enter	Display
1	1
+	1
0.1525	0.1525
=	1.1525
x^y or y^x	1.1525
4	4
=	1.764264666
×	1.764264666
500	500
=	882.1323328

The 882.132 332 8 is then rounded to $882.13.

Exploring Topic 2

Activity 1

Use the Product Law of exponents.

1. $3^2 \times 3^3 = (3\times3)\times(3\times3\times3)$
$= 3\times3\times3\times3\times3$
$= 3^5$

This implies that $3^2 \times 3^3 = 3^{2+3} = 3^5$.

It also implies that $x^a \times x^b = x^{a+b}$.

2. $(4^2)(4^3) = 4^{2+3}$
$= 4^5$

3. $(3^5)(3^9) = 3^{5+9}$
$= 3^{14}$

4. $(x^5)(x^6) = x^{5+6}$
$= x^{11}$

5. $x^2y^2 = x^2y^2$ Since the bases are not the same, you cannot simplify this any further.

6. $(2^2)(2^3)(2^4) = 2^{2+3+4}$
$= 2^9$

7. $(x^2)(x) = x^{2+1}$
$= x^3$

Activity 2

Use the Power-of-a-Power Law of exponents.

1. In this solution the base 2 has been used. The base you select will also work in the same manner.

$(2^3)^2 = (2^3)(2^3)$
$= (2\times2\times2)(2\times2\times2)$
$= 2\times2\times2\times2\times2\times2$
$= 2^6$

This example implies that $(2^3)^2 = 2^{2\times3} = 2^6$.

Therefore, $(x^a)^b = x^{a\times b} = x^{ab}$.

2. $(3^3)^4 = 3^{3\times4}$
$= 3^{12}$

3. $(5^2)^6 = 5^{2\times6}$
$= 5^{12}$

4. $(x^4)^5 = x^{4\times5}$
$= x^{20}$

5. $(x^2)^4 = x^{2\times4}$
$= x^8$

6. $(y^4)^6 = y^{4\times6}$
$= y^{24}$

7. $(10^2)^0 = 10^{2\times0}$
$= 10^0$

Activity 3

Use the Quotient Law of exponents.

1. In this solution the base 3 has been used. You can use any other base.

$$\frac{3^6}{3^2} = \frac{3\times3\times3\times3\times3\times3}{3\times3}$$

$$= \frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{3}}\times3\times3\times3\times3}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{3}}}$$

$$= 3^4$$

This implies that $\frac{3^6}{3^2} = 3^{6-2} = 3^4$.

Therefore, $\frac{x^a}{x^b} = x^{a-b}$.

2. $2^6 \div 2^2 = 2^{6-2}$

$= 2^4$

3. $7^9 \div 7^5 = 7^{9-5}$

$= 7^4$

4. $x^8 \div x^4 = x^{8-4}$

$= x^4$

5. $\frac{y^7}{y^3} = y^{7-3}$

$= y^4$

6. $x^3 \div x = x^{3-1}$ (Note that $x = x^1$.)

$= x^2$

7. $\frac{x^6y^4}{x^3y^3} = x^{6-3}y^{4-3}$

$= x^3y$

$$\frac{x^6y^4}{x^3y^3} = \frac{\overset{1}{\cancel{x}}\times\overset{1}{\cancel{x}}\times\overset{1}{\cancel{x}}\times x\times x\times x\times\overset{1}{\cancel{y}}\times\overset{1}{\cancel{y}}\times\overset{1}{\cancel{y}}\times y}{\underset{1}{\cancel{x}}\times\underset{1}{\cancel{x}}\times\underset{1}{\cancel{x}}\times\underset{1}{\cancel{y}}\times\underset{1}{\cancel{y}}\times\underset{1}{\cancel{y}}}$$

$= x^3y$

Activity 4

Use zero exponents and negative exponents.

1. This answer uses base 4 and $4^5 \div 4^5$. You may choose any other nonzero base to get the same result.

$$\frac{4^5}{4^5} = 4^{5-5} \qquad \frac{4^5}{4^5} = \frac{\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}\times\overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{4}}}$$

$$= 4^0 \qquad = \frac{1\times1\times1\times1\times1}{1\times1\times1\times1\times1}$$

$$= \frac{1}{1}$$

$$= 1$$

Therefore, $4^0 = 1$, and this implies that $x^0 = 1$.

2. You could use base 3 and $1 \div \frac{3^3}{3^7}$.

$$1 \div \frac{3^3}{3^7} = 1 \times \frac{3^7}{3^3} \qquad 1 \div \frac{3^3}{3^7} = 1 \div 3^{3-7}$$

$$= 1 \times 3^{7-3} \qquad = 1 \div 3^{-4}$$

$$= 1 \times 3^4 \qquad = \frac{1}{3^{-4}}$$

$$= 3^4$$

Therefore, $3^4 = \frac{1}{3^{-4}}$, and this implies that $x^a = \frac{1}{x^{-a}}$.

3. Use base 4 and $1 \div \frac{4^5}{4^2}$.

$$1 \div \frac{4^5}{4^2} = 1 \times \frac{4^2}{4^5} \qquad 1 \div \frac{4^5}{4^2} = 1 \div 4^{5-2}$$

$$= 1 \times 4^{2-5} \qquad = 1 \div 4^3$$

$$= 1 \times 4^{-3} \qquad = \frac{1}{4^3}$$

$$= 4^{-3}$$

Therefore $4^{-3} = \frac{1}{4^3}$, and this implies that $x^{-a} = \frac{1}{x^a}$.

4. $$\frac{3^{-5}}{4^0} = \frac{3^{-5}}{1}$$

$$= 3^{-5}$$

$$= \frac{1}{3^5}$$

5. $$\frac{y^5}{x^{-3}z^0} = \frac{y^5}{\left(x^{-3}\right)(1)}$$

$$= \frac{y^5}{x^{-3}}$$

$$= y^5 x^3$$

6. $\dfrac{(2x^9y^7)^0}{x^{-2}} = \dfrac{1}{x^{-2}}$

$= x^2$

7. $\dfrac{25^0}{4^{-2}} = \dfrac{1}{4^{-2}}$

$= 4^2$

8. $\dfrac{x^{-3}}{x^{-5}} = x^{-3-(-5)}$ or $\dfrac{x^{-3}}{x^{-5}} = \dfrac{x^5}{x^3}$

$= x^{-3+5}$ $= x^{5-3}$

$= x^2$ $= x^2$

9. $\dfrac{2^{-4}}{3^{-2}} = \dfrac{3^2}{2^4}$

Extra Help

1. a. $6^3 \times 6^2 = 6^{3+2}$

$= 6^5$

b. $5^3 \times 5^{-2} = 5^{3+(-2)}$

$= 5^{3-2}$

$= 5^1$

$= 5$

c. $x^3 \times x^5 = x^{3+5}$

$= x^8$

2. a. $\dfrac{5^4}{5^3} = 5^{4-3}$

$= 5^1$

$= 5$

b. $\dfrac{x^6}{x^2} = x^{6-2}$

$= x^4$

c. $\dfrac{x^3}{x^{-2}} = x^{3-(-2)}$

$= x^{3+2}$

$= x^5$

d. $\dfrac{x^0}{x^{-1}} = x^{0-(-1)}$

$= x^{0+1}$

$= x^1$

$= x$

3. a. $(3^2)^5 = 3^{2\times5}$

$= 3^{10}$

b. $(x^2)^7 = x^{2\times7}$

$= x^{14}$

c. $(x^{-2})^7 = x^{-2\times7}$

$= x^{-14}$ or $\dfrac{1}{x^{14}}$

Extensions

1. $\left(x^3 \times x^5\right)^3 = \left(x^{3+5}\right)^3$ or $\left(x^3 \times x^5\right)^3 = \left(x^3\right)^3 \times \left(x^5\right)^3$

$= \left(x^8\right)^3$ $= x^9 \times x^{15}$

$= x^{24}$ $= x^{9+15}$

$= x^{24}$

2. $\left(\frac{x^3}{x^5}\right)^4 = \frac{x^{12}}{x^{20}}$ or $\left(\frac{x^3}{x^5}\right)^4 = \left(x^{3-5}\right)^4$

$= x^{12-20}$ $= \left(x^{-2}\right)^4$

$= x^{-8}$ or $\frac{1}{x^8}$ $= x^{-8}$ or $\frac{1}{x^8}$

3. $\frac{\left(a^3\right) \times \left(a^2\right)^{-2}}{a^2} = \frac{a^3 \times a^{-4}}{a^2}$

$= \frac{a^{3-4}}{a^2}$

$= \frac{a^{-1}}{a^2}$

$= a^{-1-2}$

$= a^{-3}$ or $\frac{1}{a^3}$

4. $\frac{\left(a^3\right)^0 \times a^2}{\left(a^{-1}\right)^2} = \frac{1 \times a^2}{a^{-2}}$

$= \frac{a^2}{a^{-2}}$

$= a^{2-(-2)}$

$= a^{2+2}$

$= a^4$

5. $\left(\frac{y^3}{y^{-2}}\right)^4 = \frac{y^{12}}{y^{-8}}$ or $\left(\frac{y^3}{y^{-2}}\right)^4 = \left(y^{3-(-2)}\right)^4$

$= y^{12-(-8)}$ $= \left(y^{3+2}\right)^4$

$= y^{12+8}$ $= \left(y^5\right)^4$

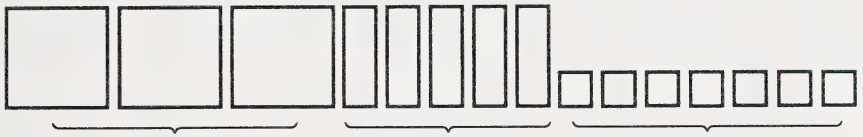
$= y^{20}$ $= y^{20}$

Exploring Topic 3

Activity 1

Add and subtract polynomials and arrange terms in alphabetical and descending order with respect to degree.

1. a. The polynomial $3x^2+5x+7$ is shown like this when tiles are used.



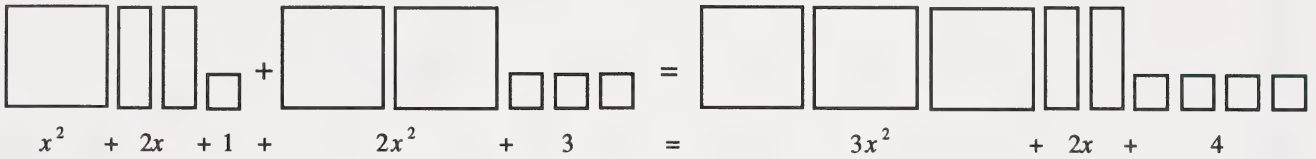
three x^2's five x's seven units

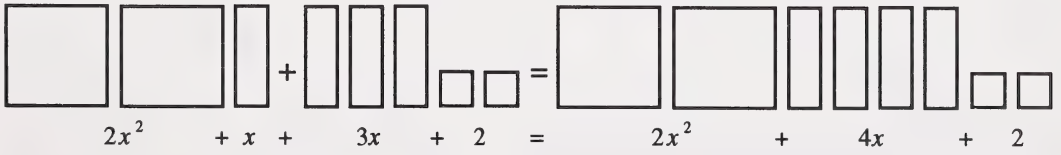
b. The polynomial x^2+x+2 is shown like this when tiles are used.

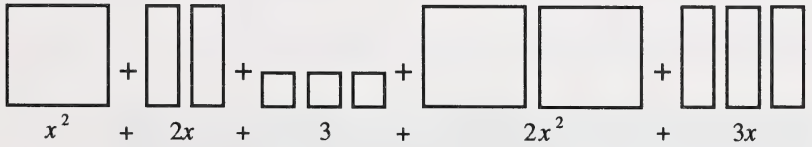
one x^2 one x two units

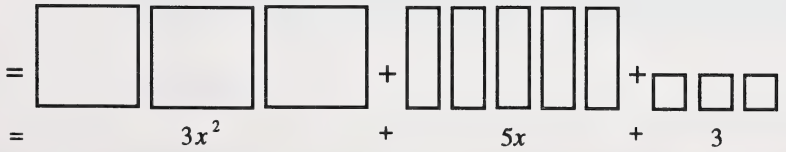
2. a. The arrangement shows the polynomial $2x^2+2x+2$.

   b. The arrangement shows the polynomial x^2+6x+1.

3. 

4. 

5. 



6. $5x+3x+7x=(5+3+7)x$

$=15x$

7. $7y^2+2y^2+6y=(7+2)y^2+6y$

$=9y^2+6y$

Notice that $6y$ and $9y^2$ are not like terms and for that reason they cannot be added to make a single term.

8. like terms

$5x^2y+3xy^2+3x^2y+2xy^2=(5+3)x^2y+(3+2)xy^2$

like terms

$=8x^2y+5xy^2$

You could rewrite $5x^2y+3xy^2+3x^2y+2xy^2$ as follows:

$5x^2y+3x^2y+3xy^2+2xy^2=(5+3)x^2y+(3+2)xy^2$

$=8x^2y+5xy^2$

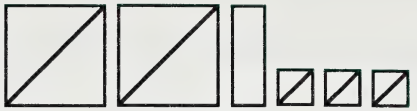
9. $6z^3y+4z^2y+2z^3y+5z^2y=(6+2)z^3y+(4+5)z^2y$

$=8z^3y+9z^2y$

You could rewrite $6z^3y+4z^2y+2z^3y+5z^2y$ as follows:

$6z^3y+2z^3y+4z^2y+5z^2y=(6+2)z^3y+(4+5)z^2y$

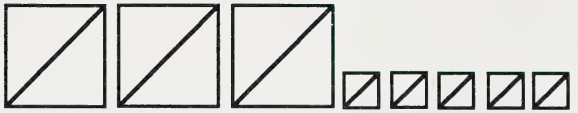
$=8z^3y+9z^2y$

10. 

$-2x^2+x-3$

11.

x^2-3x-2

12. 

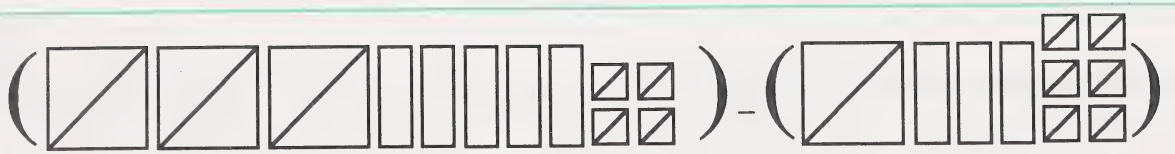
$-3x^2-5$

13.

$-x^2+2x-1$

14.

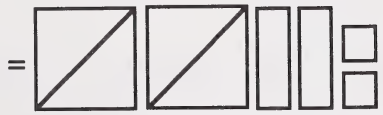
$-2x^2-4x+3$

15. a. 

$(-3x^2 + 5x - 4) - (-x^2 + 3x - 6)$

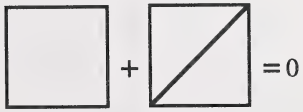
Flip the tiles over to change the sign from – to + or + to – and shade the pairs of tiles which are additive inverses.

$-3x^2 + 5x - 4 + x^2 - 3x + 6$

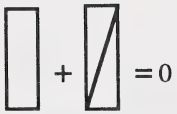


$-2x^2 + 2x + 2$

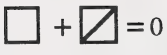
Remember what these combinations mean.



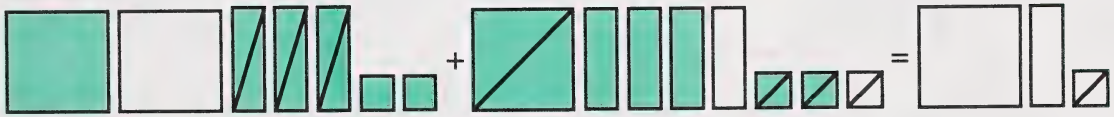
$x^2 + (-x^2) = 0$



$x + (-x) = 0$

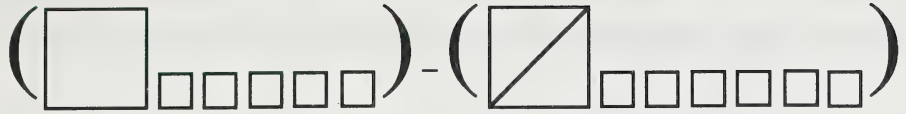


$1 + (-1) = 0$

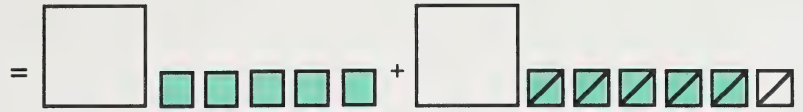
b. 

$(2x^2 - 3x + 2) + (-x^2 + 4x - 3) = x^2 + x - 1$

c.



$(x^2+5) - (-x^2+6)$

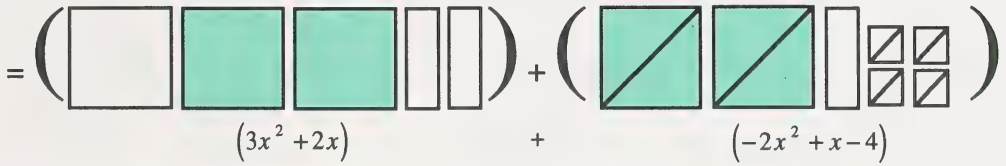


$(x^2+5) + (x^2-6)$

$2x^2-1$

d.

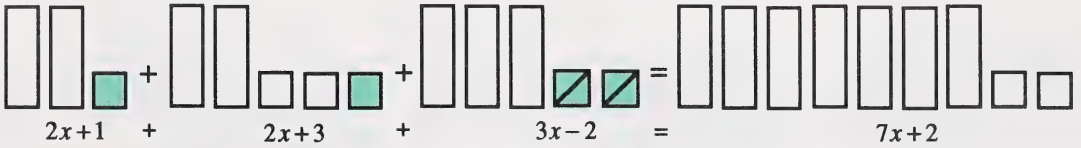
$(3x^2+2x) - (2x^2-x+4)$



$(3x^2+2x) + (-2x^2+x-4)$

x^2+3x-4

e.

f. 

16. a. $(3x^2-2x)+(4x-x^2)=3x^2-2x+4x-x^2$

$=3x^2-x^2-2x+4x$

$=(3-1)x^2+(-2+4)x$

$=2x^2+2x$

b. $(4y^3x-3y^2x^2)-(2x^2+3y^3x-4y^2x^2)=4y^3x-3y^2x^2-2x^2-3y^3x+4y^2x^2$

$=4y^3x-3y^3x-3y^2x^2+4y^2x^2-2x^2$

$=(4-3)y^3x+(-3+4)y^2x^2-2x^2$

$=y^3x+y^2x^2-2x^2$

c. $2x^2-\left(3y^2x^2-4yx^2-5x^2\right)=2x^2-3y^2x^2+4yx^2+5x^2$

$=2x^2+5x^2-3y^2x^2+4yx^2$

$=(2+5)x^2-3y^2x^2+4yx^2$

$=7x^2-3y^2x^2+4yx^2$

d. $3x^2y-5xy^2+3y^3-\left(-3x^2y+2xy^2-2y^3\right)=3x^2y-5xy^2+3y^3+3x^2y-2xy^2+2y^3$

$=3x^2y+3x^2y-5xy^2-2xy^2+3y^3+2y^3$

$=(3+3)x^2y+(-5-2)xy^2+(3+2)y^3$

$=6x^2y+\left(-7xy^2\right)+5y^3$

17. a. $\left(4x^2-5x+3\right)-\left(6x-5x^2+8\right)=$

$$\begin{array}{rcl} 4x^2-5x+3 & = & 4x^2-5x+3 \\ \underline{-\left(-5x^2+6x+8\right)} & = & +\underline{5x^2-6x-8} \\ & & 9x^2-11x-5 \end{array}$$

b. $\left(x^2-2xy-y^2\right)-\left(4y^2-8xy-3x^2\right)=$

$$\begin{array}{rcl} x^2-2xy-y^2 & = & x^2-2xy-y^2 \\ \underline{-\left(-3x^2-8xy+4y^2\right)} & = & +\underline{3x^2+8xy-4y^2} \\ & & 4x^2+6xy-5y^2 \end{array}$$

c. $(2x^3-3x^2)+(4x^2-3x^3)-(4x^3-5x^2+3x)=$

$$\begin{array}{rcl} 2x^3-3x^2 & = & 2x^3-3x^2 \\ +(-3x^3+4x^2) & = & -3x^3+4x^2 \\ \underline{-(+4x^3-5x^2+3x)} & = & \underline{-4x^3+5x^2-3x} \\ & & -5x^3+6x^2-3x \end{array}$$

18. a. $P=a+b+c$

$P=(2x-y)+(x+2y)+(x+y)$

$$\begin{array}{r} 2x-y \\ x+2y \\ +\ x+y \\ \hline 4x+2y \end{array}$$

The perimeter of ΔABC is $(4x+2y)$ units.

b. $P=AB+BC+CD+AD$

$P=(6x+4y-y^2)+(y^2+x)+(5x^2+3y)+(2x^2-y)$

$$\begin{array}{r} 6x+4y-y^2 \\ x\quad +y^2 \\ 5x^2\quad +3y \\ +\ 2x^2\quad -y \\ \hline 7x^2+7x+6y \end{array}$$

The perimeter of quadrilateral $ABCD$ is $(7x^2+7x+6y)$ units.

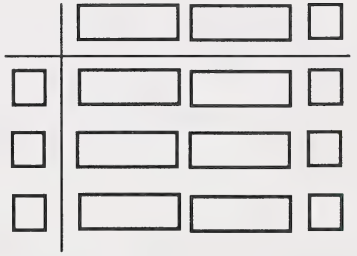
Activity 2

Multiply polynomials by monomials.

1.

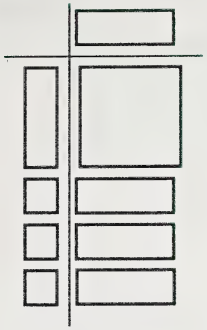
Therefore, $2x(3x+2)=6x^2+4x$.

2.



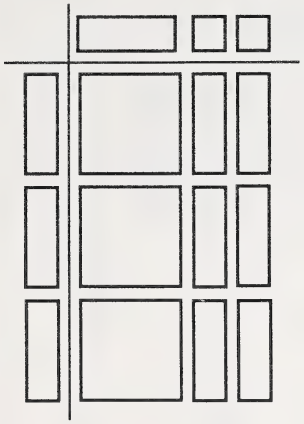
Therefore, $3(2x+1)=6x+3$.

3.



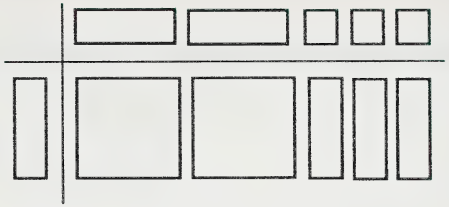
Therefore, $(x+3)(x) = x^2 + 3x$.

4.



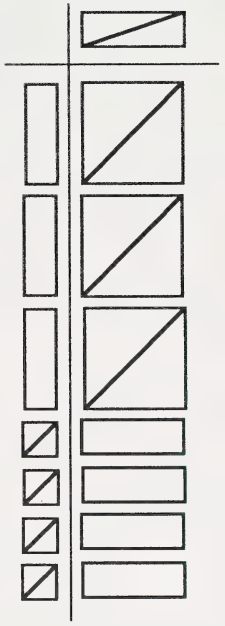
Therefore, $3x(x+2) = 3x^2 + 6x$.

5.



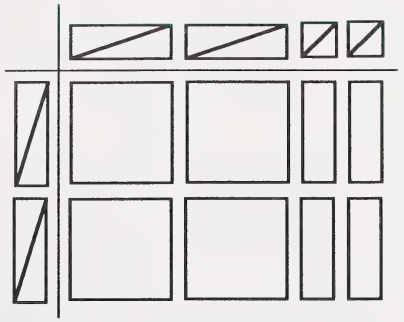
Therefore, $x(2x+3) = 2x^2 + 3x$.

6.



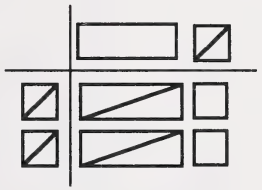
Therefore, $(3x-4)(-x) = -3x^2 + 4x$.

7.



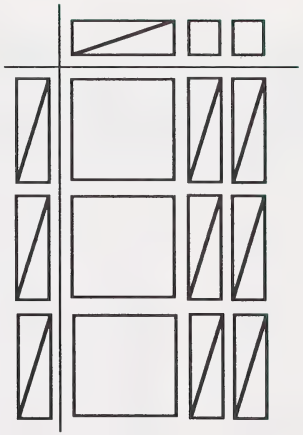
Therefore, $(-2x)(-2x-2)=4x^2+4x$.

8.



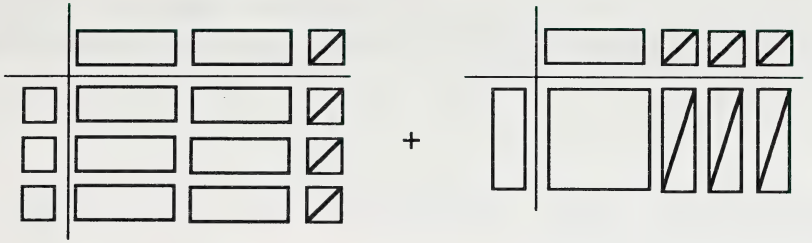
Therefore, $-2(x-1)=-2x+2$.

9.



Therefore, $-3x(-x+2)=3x^2-6x$.

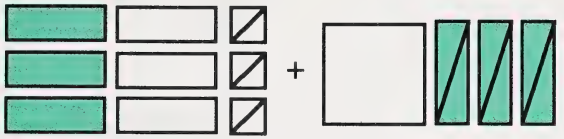
10.



Therefore, $3(2x-1)+x(x-3)=(6x-3)+\left(x^2-3x\right)$

$$=x^2+3x-3.$$

To get x^2+3x-3 from $(6x-3)+\left(x^2-3x\right)$, use the tiles and the zero property to get the following:



Notice how the shaded tiles cancel.

11. a. $(3x^2y^3)(2xy^4) = (3)(2)(x^2)(x)(y^3)(y^4)$

$= 6x^{2+1}y^{3+4}$

$= 6x^3y^7$

b. $(-3x^3y^5)(-2xy^4)(4x^2y) = (-3)(-2)(4)(x^3)(x^1)(x^2)(y^5)(y^4)(y^1)$

$= 24x^{3+1+2}y^{5+4+1}$

$= 24x^6y^{10}$

12. a. $2y(3x^2 - y^3) = 6yx^2 - 2y^4$

$= 6x^2y - 2y^4$

b. $-x(y-3x) = -xy + 3x^2$

$= 3x^2 - xy$

c. $(-2z)(-3z)(2x^2z - 3xz^3) = (6z^2)(2x^2z - 3xz^3)$ (Multiply $-2z$ and $-3z$ first.)

$= 12x^2z^3 - 18xz^5$ (Use the distributive property.)

13. The degree of $6x^3y^2 - 5xy^3 + 6$ is 5 because $6x^3y^2$ is the term with the highest degree. The degrees of the other two terms are 4 and 0, respectively.

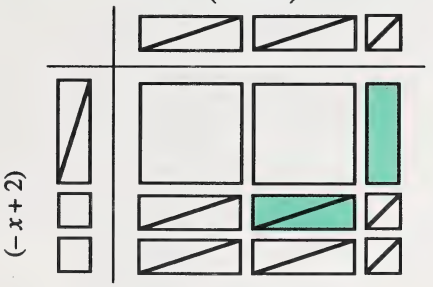
14. The degree of $7x^4y^7$ is 11 because the degree of the term is the sum of the exponents 4 and 7.

15. The degree of y in the term $8x^7y^6z^4$ is 6 because the exponent of y is 6.

Activity 3

Multiply binomials by binomials.

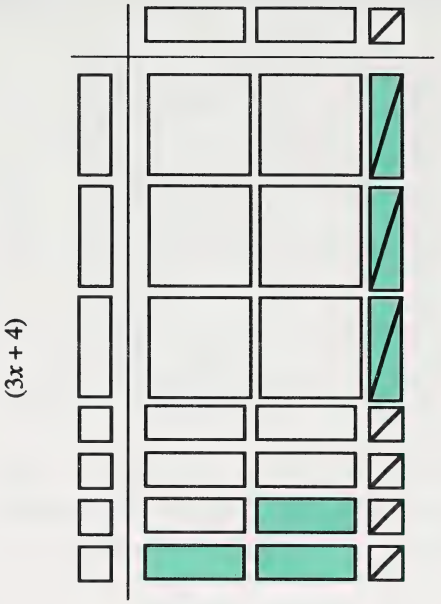
1. $(-2x-1)$



$$(-x+2)(-2x-1)=2x^2+x+-4x+-2$$
$$=2x^2-3x-2$$

The shading indicates that these tiles may be removed because $+x+(-x)=0$.

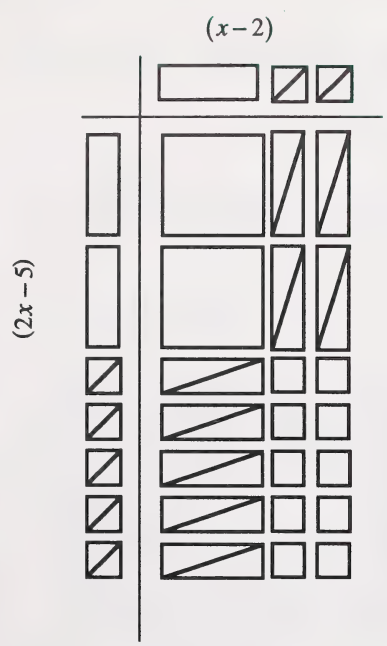
2. $(2x-1)$



$$(3x+4)(2x-1)=6x^2+8x+-3x-4$$
$$=6x^2+5x-4$$

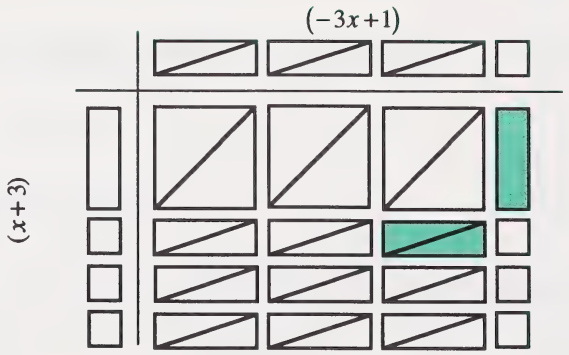
Notice how and why certain tiles are cancelled or removed here.

3.



$$(2x-5)(x-2)=2x^2-4x-5x+10$$
$$=2x^2-9x+10$$

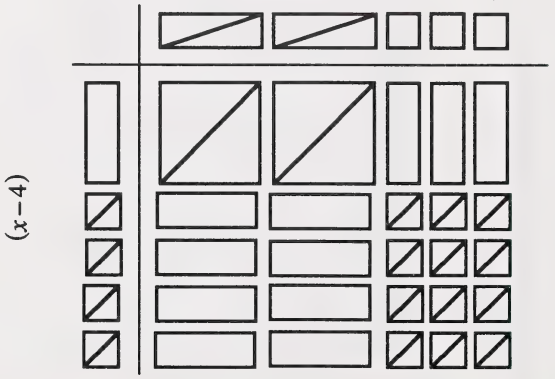
4.



$$(x+3)(-3x+1)=-3x^2+x-9x+3$$
$$=-3x^2-8x+3$$

5. To avoid confusion, change $(3-2x)$ to $(-2x+3)$. These are the same since addition can be done in any order you choose.

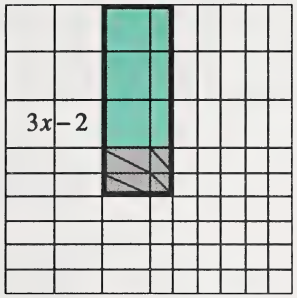
$(3-2x)$ or $(-2x+3)$



$$(x-4)(3-2x) = -2x^2 + 3x + 8x - 12$$
$$= -2x^2 + 11x - 12$$

6. Use a binomial grid.

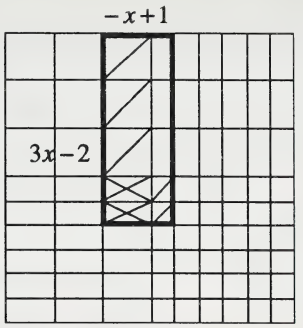
For $(3x-2)(-x+1)$ begin with the following diagram.



Work with $(3x-2)$ first.

Mark the positive because the $3x$ is positive.

Mark the negative because the -2 is negative.

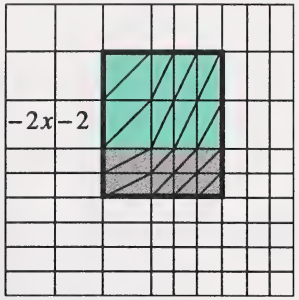


Notice that when you multiply everything in the first column by $-x$, all of the values change; the three x^2's become $-x^2$'s and the two $-x$'s become $+x$'s.

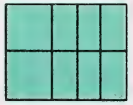
When you multiply everything in the second column by $+1$, all of the values remain the same.

$$\therefore (3x-2)(-x+1) = -3x^2 + 2x + 3x - 2$$
$$= -3x^2 + 5x - 2$$

7. For $(-2x-2)(x-3)$ begin with the following diagram.

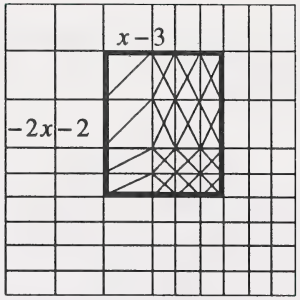


Work with $-2x-2$ first.

Mark the  negative because of $-2x$ being negative.

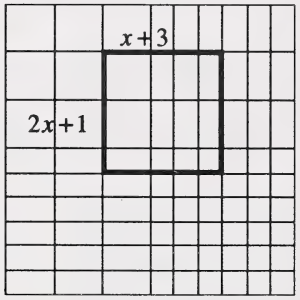
Mark the negative because of -2 being negative.

When you multiply everything in the first column by $+x$, all of the values in the first column remain the same. When the last three columns are multiplied by -3, all parts of the last three columns change.

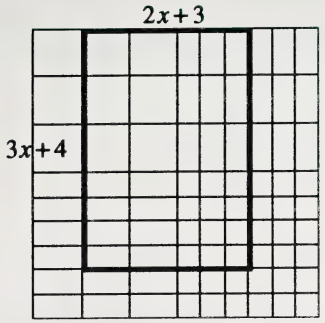


$$\therefore (-2x-2)(x-3) = -2x^2+6x-2x+6$$
$$= -2x^2+4x+6$$

Notice that two of plus six of equal four of .

8. 

$$\therefore (2x+1)(x+3) = 2x^2+x+6x+3$$
$$= 2x^2+7x+3$$

9. 

$\therefore (3x+4)(2x+3) = 6x^2 + 9x + 8x + 12$

$= 6x^2 + 17x + 12$

10. $(x+2)(x+2) = x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

11. $(y+3)(y+3) = y^2 + 3y + 3y + 9$

$= y^2 + 6y + 9$

12. $(2x+1)(x-3) = 2x^2 - 6x + x - 3$

$= 2x^2 - 5x - 3$

13. $(x+2)(3x-5) = 3x^2 - 5x + 6x - 10$

$= 3x^2 + x - 10$

14. $(3-x)(x+4) = 3x + 12 - x^2 - 4x$

$= -x^2 - x + 12$

15. $(x-1)(x-1) = x^2 - x - x + 1$

$= x^2 - 2x + 1$

16. $(2-y)(2-y) = 4 - 2y - 2y + y^2$

$= y^2 - 4y + 4$

17. $(x+2)(x-2) = x^2 - 2x + 2x - 4$

$= x^2 - 4$

18. $(3-y)(3+y) = 9 + 3y - 3y - y^2$

$= -y^2 + 9$

19. $(2x-1)(2x+1) = 4x^2 + 2x - 2x - 1$

$= 4x^2 - 1$

20. $(3x+2y)(2x-3y) = 6x^2 - 9xy + 4xy - 6y^2$

$= 6x^2 - 5xy - 6y^2$

21. $(6y-8x)(4y-7x) = 24y^2 - 42xy - 32xy + 56x^2$

$= 24y^2 - 74xy + 56x^2$

$= 56x^2 - 74xy + 24y^2$

All of these answers have been put in descending order of powers and in alphabetical order.

Did you remember to multiply each term in one polynomial by all the terms in the other?

Activity 4

Divide polynomials by monomials.

1. a. $\frac{4x-2}{2} = \frac{4x}{2} - \frac{2}{2}$

$= 2x - 1$

b. $\frac{27x^3y + 9xy^3}{3xy} = \frac{27x^3y}{3xy} + \frac{9xy^3}{3xy}$

$= 9x^2 + 3y^2$

c. $\frac{14a^2b - 7ab^2 + 21b^3}{7b} = \frac{14a^2b}{7b} - \frac{7ab^2}{7b} + \frac{21b^3}{7b}$

$= 2a^2 - ab + 3b^2$

d. $\frac{6x^3y - 9x^2y^2 + 12xy^3}{-3xy} = \frac{6x^3y}{-3xy} + \frac{-9x^2y^2}{-3xy} + \frac{12xy^3}{-3xy}$

$= -2x^2 + 3xy - 4y^2$

2. $A = lw$

or

$lw = A$

$\frac{lw}{w} = \frac{A}{w}$

$l = \frac{A}{w}$

$\therefore l = \frac{6x^2 + 4xy}{2x}$

$= \frac{6x^2}{2x} + \frac{4xy}{2x}$

$= 3x + 2y$

Check:

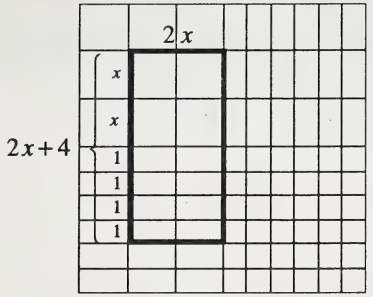
$A = lw$

$6x^2 + 4xy = (3x + 2y)(2x)$

$6x^2 + 4xy = (3x)(2x) + (2y)(2x)$

$6x^2 + 4xy = 6x^2 + 4xy$

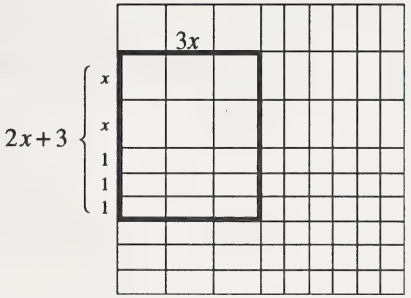
3. The area of the rectangle is $4x^2 + 8x$.



After drawing the rectangle with an area of $4x^2 + 8x$ and a width of $2x$, you can see that the length would be $2x + 4$.

$\therefore (4x^2 + 8x) + 2x = 2x + 4$

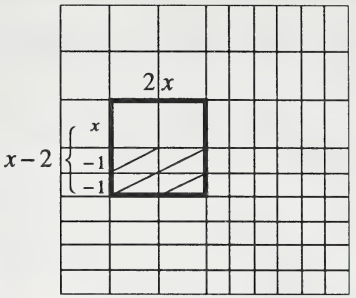
4. The area of the rectangle is $6x^2 + 9x$.



You can see that the length would be $2x + 3$.

Therefore, $\frac{6x^2 + 9x}{3x} = 2x + 3$.

5. The area of the rectangle is $2x^2 - 4x$.



You can see that the length would be $x - 2$.

Therefore, $\frac{2x^2 - 4x}{2x} = x - 2$.

Extra Help

1. $(-2)(x+1) = (-2)(x) + (-2)(+1)$
$= -2x + (-2)$
$= -2x - 2$

2. $(x)(x-3) = (x)(x) + (x)(-3)$
$= x^2 + (-3x)$
$= x^2 - 3x$

3. $x^2(x-2) = (x^2)(x) + (x^2)(-2)$

$= x^3 + (-2x^2)$

$= x^3 - 2x^2$

You must remember that $(x^2)(x) = x^{2+1} = x^3$.

4. $(x+1)(5) = (x)(5) + (1)(5)$

$= 5x + 5$

5. $(2a)(a^2 - 3a) = (2a)(a^2) + (2a)(-3a)$

$= 2a^3 + (-6a^2)$

$= 2a^3 - 6a^2$

6. $(b-2)(b^2+3) = (b)(b^2) + (b)(3) + (-2)(b^2) + (-2)(3)$

$= b^3 + 3b - 2b^2 - 6$

$= b^3 - 2b^2 + 3b - 6$

7. $(c+b)(c+b) = (c)(c) + (c)(b) + (b)(c) + (b)(b)$

$= c^2 + cb + cb + b^2$

$= c^2 + 2cb + b^2$

or

$(b+c)(b+c) = b^2 + 2bc + c^2$

Note that $c^2 + 2cb + b^2$ would be in descending alphabetical order while $b^2 + 2bc + c^2$ would be in ascending alphabetical order.

8. $(2a-1)(a+3) = (2a)(a) + (2a)(3) + (-1)(a) + (-1)(3)$

$= 2a^2 + 6a - a - 3$

$= 2a^2 + 5a - 3$

9.
$$
\begin{array}{r}
2x^2 - 3x + 1 \\
3x \overline{)\,6x^3 - 9x^2 + 3x} \\
\underline{6x^3} \\
0 - 9x^2 \\
\underline{-9x^2} \\
0 + 3x \\
\underline{+3x} \\
0
\end{array}
$$

Check: $3x(2x^2 - 3x + 1) = 6x^3 - 9x^2 + 3x$

10.
$$-3x^2 \overline{)9x^6 - 6x^4 - 18x^2}\quad -3x^4 + 2x^2 + 6$$
$$\begin{array}{r} 9x^6 \\ \hline 0 \quad -6x^4 \\ \quad -6x^4 \\ \hline 0 \quad -18x^2 \\ \quad -18x^2 \\ \hline 0 \end{array}$$

Check: $-3x^2\left(-3x^4 + 2x^2 + 6\right) = 9x^6 - 6x^4 - 18x^2$

11.
$$2xy \overline{)16x^3y^3 - 8x^2y^2 + 4xy}\quad 8x^2y^2 - 4xy + 2$$
$$\begin{array}{r} 16x^3y^3 \\ \hline 0 \quad -8x^2y^2 \\ \quad -8x^2y^2 \\ \hline 0 \quad +4xy \\ \quad +4xy \\ \hline 0 \end{array}$$

Check: $2xy\left(8x^2y^2 - 4xy + 2\right) = 16x^3y^3 - 8x^2y^2 + 4xy$

Extensions

1.
$$x - 2 \overline{)x^2 - 5x + 6}\quad x - 3$$
$$\begin{array}{r} x^2 - 2x \\ \hline \quad (-3x) + 6 \\ \quad -3x + 6 \\ \hline 0 \end{array}$$

Be careful when subtracting $-2x$ from $-5x$.

$-5x - (-2x) = -5x + 2x$

$= (-3x)$

2.
$$x - 3 \overline{)2x^2 - 5x - 3}\quad 2x + 1$$
$$\begin{array}{r} 2x^2 - 6x \\ \hline \quad (x) - 3 \\ \quad x - 3 \\ \hline 0 \end{array}$$

$-5x - (-6x) = -5x + 6x$

$= (x)$

3.
$$-x + 2 \overline{)x^3 - 4x^2 + 7x - 6}\quad -x^2 + 2x - 3$$
$$\begin{array}{r} (x^3) - 2x^2 \\ \hline \quad -2x^2 + 7x \\ \quad -2x^2 + 4x \\ \hline \quad 3x - 6 \\ \quad 3x - 6 \\ \hline 0 \end{array}$$

$(-x^2)(-x) = (x^3)$

4.
$$\begin{array}{r} 3x^2 - 4x + 5 \\ 2x-1\overline{)\,6x^3 - 11x^2 + 14x - 5} \\ \underline{6x^3 - 3x^2} \\ -8x^2 + 14x \\ \underline{-8x^2 + 4x} \\ 10x - 5 \\ \underline{10x - 5} \\ 0 \end{array}$$

Exploring Topic 4

Activity 1

Factor polynomials with a common factor.

1. $3x+9=3(x+3)$

   Check: $3(x+3)=3(x)+3(3)$
   $=3x+9$

2. $8y-6=2(4y-3)$

   Check: $2(4y-3)=2(4y)+2(-3)$
   $=8y-6$

3. $x^2+xy=x(x+y)$

   Check: $x(x+y)=x(x)+x(y)$
   $=x^2+xy$

4. $ax^2+ay=a\left(x^2+y\right)$

   Check: $a\left(x^2+y\right)=a\left(x^2\right)+a(y)$
   $=ax^2+ay$

5. $21x^2+14x+7=7\left(3x^2+2x+1\right)$

   Check: $7\left(3x^2+2x+1\right)=7\left(3x^2\right)+7(2x)+7(1)$
   $=21x^2+14x+7$

6. $4x^3y+8x^2y^2-4xy^3=4xy\left(x^2+2xy-y^2\right)$

   Check: $4xy\left(x^2+2xy-y^2\right)=4xy\left(x^2\right)+4xy(2xy)+4xy\left(-y^2\right)$
   $=4x^3y+8x^2y^2-4xy^3$

7. $6x^4y^4 + 9x^3y^3 - 3x^2y^2 = 3x^2y^2\left(2x^2y^2 + 3xy - 1\right)$

Check: $3x^2y^2\left(2x^2y^2 + 3xy - 1\right) = 3x^2y^2\left(2x^2y^2\right) + 3x^2y^2\,(3xy) + 3x^2y^2\,(-1)$

$= 6x^4y^4 + 9x^3y^3 - 3x^2y^2$

8. Area $= \left(12x^2 - 36x\right)$ square units

Length $= 4x$ units

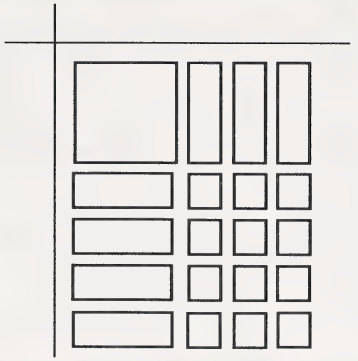
When you take the factor $4x$ out of $12x^2 - 36x$, you get $3x - 9$. Therefore, the length of the other side is $(3x - 9)$ units.

Check: $4x(3x - 9) = 12x^2 - 36x$

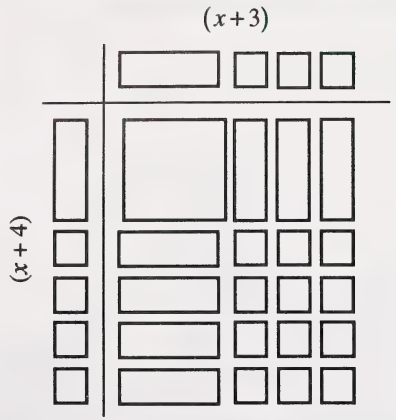
Activity 2

Factor trinomials of the form $ax^2 + bx + c$, where $a = 1$.

1. To factor $x^2 + 7x + 12$, start with four and three x tiles because 4 and 3 are factors of 12 which add to 7.

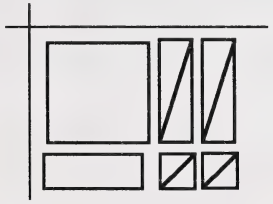


Complete as shown here.

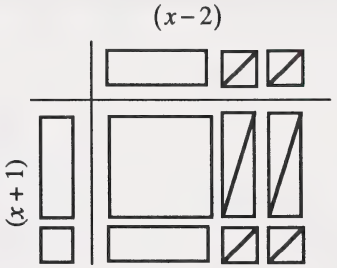


$\therefore x^2 + 7x + 12 = (x+3)(x+4)$

2. To factor $x^2 - x - 2$, start with this.



Complete as shown here.

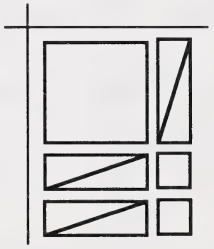


$\therefore x^2 - x - 2 = (x+1)(x-2)$

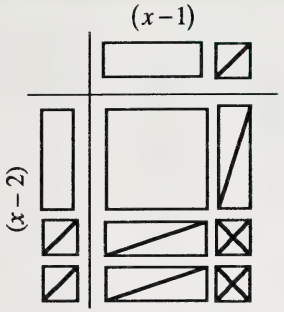
You should notice here that two plus one equal one

or $-2x + (+x) = -x$.

3. To factor $x^2 - 3x + 2$, start with this.



Complete as shown here.

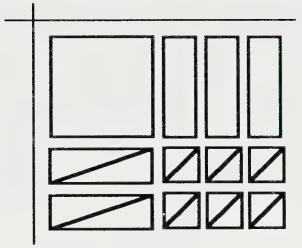


$\therefore x^2 - 3x + 2 = (x-2)(x-1)$

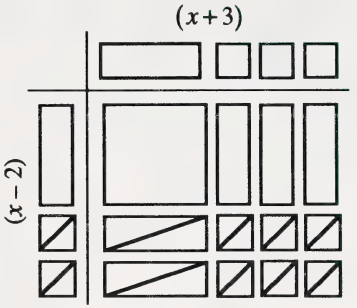
A flat like ▭ is positive. A flat like ▨ is negative. From this it must follow that ▨ × ▨ = ▩ or ▭ since $(-)\times(-) = +$. Therefore, a flat like ▭ is positive and a flat like ▩ is positive. The same applies to the other shapes where

× = or and ▨ × ▨ = ▩ or ▭ .

4. To factor $x^2 + x - 6$, start with this.

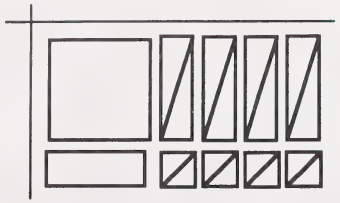


Complete as shown here.

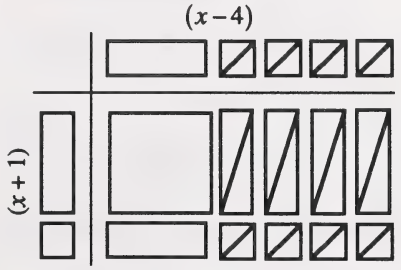


$\therefore x^2 + x - 6 = (x-2)(x+3)$

5. To factor $x^2 - 3x - 4$, start with this.

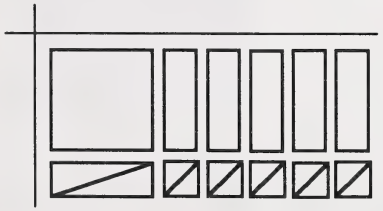


Complete as shown here.

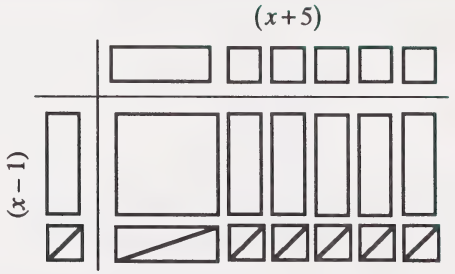


$\therefore x^2 - 3x - 4 = (x+1)(x-4)$

6. To factor $x^2 + 4x - 5$, start with this.

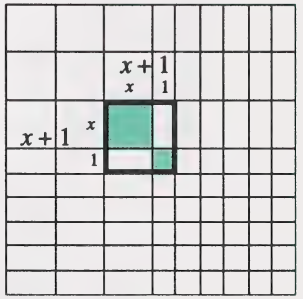


Complete as shown here.



$\therefore x^2 + 4x - 5 = (x-1)(x+5)$

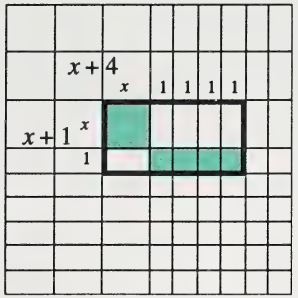
7. Factor $x^2 + 2x + 1$.



Begin a rectangle by placing the x^2 flat in one corner and the unit tile in the other corner. Complete the rectangle and count the terms along two adjacent sides.

Therefore, $x^2 + 2x + 1 = (x+1)(x+1)$.

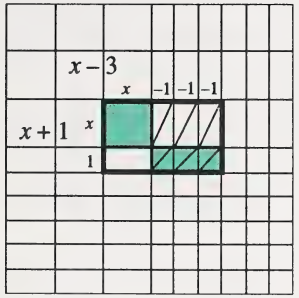
8. Factor x^2+5x+4.



Begin a rectangle by placing the x^2 flat in one corner and four unit tiles in the other corner. Complete the rectangle and count the terms along two adjacent sides.

Therefore, $x^2+5x+4=(x+4)(x+1)$.

9. Factor x^2-2x-3.



Begin a rectangle by placing the x^2 flat in one corner and the three negative unit tiles in the other corner. Complete the rectangle and count the terms along two adjacent sides.

Therefore, $x^2-2x-3=(x-3)(x+1)$.

10. Since $6+7=13$ and $6\times7=42$,
$(x+7)(x+6)=x^2$ **+13** x **+42** .

11. Since $4+(-4)=0$ and $4\times(-4)=-16$,
$(x+4)(x-4)=x^2$ **+0** x **−16** .

12. Since $-2+(-3)=-5$ and $-2\times(-3)=6$,
$(x-2)(x-3)=x^2$ **−5** x **+6** .

13. Since $-8+(+6)=-2$ and $-8\times(+6)=-48$,
$(x-8)(x+6)=x^2$ **−2** x **−48** .

14. Since $-3+(+5)=+2$ and $-3\times(+5)=-15$,
$(y-3)(y+5)=y^2$ **+2** y **−15** .

15. Since $-9+(-9)=-18$ and $-9\times(-9)=81$,
$(z-9)(z-9)=z^2$ **−18** z **+81** .

16. $x^2+11x+10=(x+10)(x+1)$

17. $x^2+6x+8=(x+4)(x+2)$

18. $x^2+7x+12=(x+4)(x+3)$

19. $x^2-2x-15=(x-5)(x+3)$

20. $x^2 - 9x + 18 = (x-6)(x-3)$

21. $x^2 - 4x + 4 = (x-2)(x-2)$ or $(x-2)^2$

22. $x^2 - 5x - 6 = (x-6)(x+1)$

23. $y^2 - 3y - 4 = (y-4)(y+1)$

24. $z^2 - 9z + 20 = (z-5)(z-4)$

25. $a^2 + 2a - 3 = (a+3)(a-1)$

26. $7x^2 - 21x + 14 = 7\left(x^2 - 3x + 2\right)$
$= 7(x-2)(x-1)$

27. $3b^2 + 18b - 48 = 3\left(b^2 + 6b - 16\right)$
$= 3(b+8)(b-2)$

28. $2y^2 - 14y + 20 = 2\left(y^2 - 7y + 10\right)$
$= 2(y-5)(y-2)$

29. $4x^2 + 16x + 12 = 4\left(x^2 + 4x + 3\right)$
$= 4(x+3)(x+1)$

30. $5a^2 - 20a + 20 = 5\left(a^2 - 4a + 4\right)$
$= 5(a-2)(a-2)$ or $5(a-2)^2$

31. $2x^2 + 6x - 56 = 2\left(x^2 + 3x - 28\right)$
$= 2(x+7)(x-4)$

32. $5x^2 + 10x - 75 = 5\left(x^2 + 2x - 15\right)$
$= 5(x+5)(x-3)$

33. $2b^2 - 16b - 18 = 2\left(b^2 - 8b - 9\right)$
$= 2(b-9)(b+1)$

34. $x^3 - 6x^2 + 8x = x\left(x^2 - 6x + 8\right)$
$= x(x-4)(x-2)$

35. $3z^2 - 15z - 42 = 3\left(z^2 - 5z - 14\right)$
$= 3(z-7)(z+2)$

Activity 3

Recognize and factor perfect square trinomials.

1. Yes, $x^2 - 8x + 16$ is a perfect square trinomial.

$$x^2 - 8x + 16 = (x-4)(x-4)$$
$$= (x-4)^2$$

2. Yes, $x^2 + 10x + 25$ is a perfect square trinomial.

$$x^2 + 10x + 25 = (x+5)(x+5)$$
$$= (x+5)^2$$

3. Yes, $y^2 + 4y + 4$ is a perfect square trinomial.

$$y^2 + 4y + 4 = (y+2)(y+2)$$
$$= (y+2)^2$$

4. Yes, $x^2 - 2x + 1$ is a perfect square trinomial.

$$x^2 - 2x + 1 = (x-1)(x-1)$$
$$= (x-1)^2$$

5. Yes, $x^2 + 5x + \frac{25}{4}$ is a perfect square trinomial.

$$x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right)$$
$$= \left(x + \frac{5}{2}\right)^2$$

6. No, $2x^2 - 12x + 18$ is not a trinomial square. However, once the common factor 2 is factored out, the remaining trinomial is a perfect square.

$$2x^2 - 12x + 18 = 2\left(x^2 - 6x + 9\right)$$
$$= 2(x-3)(x-3)$$
$$= 2(x-3)^2$$

7. Yes, $(x+2)^2 + 4(x+2) + 4$ is a perfect square trinomial.

In $(x+2)^2 + 4(x+2) + 4$ let $(x+2) = y$. Now rewrite the trinomial as $y^2 + 4y + 4$.

Factor $y^2 + 4y + 4$.

$$y^2 + 4y + 4 = (y+2)(y+2)$$
$$= (y+2)^2$$

Now substitute $(x + 2)$ for y.

$$(y+2)^2 = (x+2+2)^2$$
$$= (x+4)^2$$

Another method for factoring this trinomial is as follows:

Expand $(x+2)^2+4(x+2)+4$ first.

$$(x+2)^2+4(x+2)+4 = x^2+4x+4+4x+8+4$$
$$= x^2+8x+16$$

Now factor $x^2+8x+16$.

$$x^2+8x+16 = (x+4)(x+4)$$
$$= (x+4)^2$$

8. No, $3x^2-18x+27$ is not a trinomial square. First factor out the common factor of 3. The remaining polynomial is a perfect square trinomial.

$$3x^2-18x+27 = 3\left(x^2-6x+9\right)$$
$$= 3(x-3)(x-3)$$
$$= 3(x-3)^2$$

9. No, $x^2+10x+16$ is not a perfect square trinomial.
$x^2+10x+16 = (x+8)(x+2)$

10. No, x^2+25 is not a perfect square trinomial. This polynomial is not factorable.

Activity 4

Recognize and factor a difference of squares.

1. Yes, x^2-25 is a difference of squares.
$x^2-25 = (x+5)(x-5)$

2. No, y^2+16 is not a difference of squares.
The polynomial y^2+16 cannot be factored.

3. No, x^2-4x-4 is not a difference of squares.
The polynomial x^2-4x-4 cannot be factored.

4. Yes, x^2-9 is a difference of squares.
$x^2-9 = (x+3)(x-3)$

5. Yes, $x^2-\frac{9}{4}$ is a difference of squares.
$x^2-\frac{9}{4} = \left(x+\frac{3}{2}\right)\left(x-\frac{3}{2}\right)$

6. Yes, x^2-144 is a difference of squares.
$x^2-144 = (x+12)(x-12)$

7. Yes, $x^2 - y^2$ is a difference of squares.

$x^2 - y^2 = (x+y)(x-y)$

8. Yes, $16x^4 - 9y^8$ is a difference of squares.

$16x^4 - 9y^8 = \left(4x^2 + 3y^4\right)\left(4x^2 - 3y^4\right)$ since $\sqrt{16x^4} = 4x^2$

and $\sqrt{9y^8} = 3y^4$

9. Yes, $(x-2)^2 - (y+2)^2$ is a difference of squares.

$$\begin{aligned}(x-2)^2 - (y+2)^2 &= [(x-2)+(y+2)][(x-2)-(y+2)] \\ &= [x-2+y+2][x-2-y-2] \\ &= (x+y-2+2)(x-y-2-2) \\ &= (x+y)(x-y-4)\end{aligned}$$

Another method is as follows:

Let $(x-2) = a$ and let $(y+2) = b$.

Substitute to get $(x-2)^2 - (y+2)^2 = a^2 - b^2$.

Now factor $a^2 - b^2$.

$a^2 - b^2 = (a+b)(a-b)$

Now substitute $a = (x-2)$ and $b = (y+2)$ in $(a+b)(a-b)$.

$$\begin{aligned}(a+b)(a-b) &= [(x-2)+(y+2)][(x-2)-(y+2)] \\ &= [x-2+y+2][x-2-y-2] \\ &= (x+y-2+2)(x-y-2-2) \\ &= (x+y)(x-y-4)\end{aligned}$$

Activity 5

Factor polynomials which require a combination of methods in order to be fully factored.

1. $8x^2 - 4 = 4\left(2x^2 - 1\right)$

Note that $\left(2x^2 - 1\right)$ cannot be factored further.

2. $2x^2y - 16 = 2\left(x^2y - 8\right)$

Note that $\left(x^2y - 8\right)$ cannot be factored further.

3. $3x^2y^2 - 9xy^2 + 6y^2 = 3y^2\left(x^2 - 3x + 2\right)$

$= 3y^2(x-2)(x-1)$

4. $2y(x+3) - 4(x+3) = (x+3)(2y-4)$

$= (x+3)(2)(y-2)$

$= 2(x+3)(y-2)$

Notice that here $(x+3)$ can be factored out of $2y(x+3)$ and $-4(x+3)$.

5. $3a(x^2-2)-9a = 3a[(x^2-2)-3]$

$= 3a(x^2-2-3)$

$= 3a(x^2-5)$

Notice that here $3a$ can be taken out of $3a(x^2-2)$ and $-9a$.

6. $x^2(2x-1)-3x(2x-1)-10(2x-1) = (2x-1)(x^2-3x-10)$

$= (2x-1)(x-5)(x+2)$

Notice that here $2x-1$ can be taken out of all three terms.

Extra Help

1. $x^2+2x-15$

$-15x^2$ | $2x$ $\qquad$ $-15x^2$ | $5x$, $-3x$ | $2x$

$\therefore x^2+2x-15 = x^2+5x-3x-15$

$= (x^2+5x)-(3x+15)$

$= x(x+5)-3(x+5)$

$= (x-3)(x+5)$

2. $x^2+3x-10$

$-10x^2$ | $3x$ $\qquad$ $-10x^2$ | $5x$, $-2x$ | $3x$

$\therefore x^2+3x-10 = x^2+5x-2x-10$

$= (x^2+5x)-(2x+10)$

$= x(x+5)-2(x+5)$

$= (x-2)(x+5)$

3. $x^2-8x+12$

$12x^2$ | $-8x$ $\qquad$ $12x^2$ | $-6x$, $-2x$ | $-8x$

$x^2-8x+12 = x^2-6x-2x+12$

$= (x^2-6x)-(2x-12)$

$= x(x-6)-2(x-6)$

$= (x-2)(x-6)$

4. $x^2+14x+49$

$49x^2$		$49x^2$	$7x$
			$7x$
	$14x$		$14x$

$$\begin{aligned}\therefore x^2+14x+49 &= x^2+7x+7x+49\\ &=\left(x^2+7x\right)+(7x+49)\\ &=x(x+7)+7(x+7)\\ &=(x+7)(x+7)\\ &=(x+7)^2\end{aligned}$$

5. x^2-y^2

$-x^2y^2$		$-x^2y^2$	xy
			$-xy$
	0		0

$$\begin{aligned}\therefore x^2-y^2 &= x^2+xy-xy-y^2\\ &=\left(x^2+xy\right)-\left(xy+y^2\right)\\ &=x(x+y)-y(x+y)\\ &=(x-y)(x+y)\end{aligned}$$

Extensions

1. a. $$\begin{aligned}\frac{x^2-2x+1}{x-1} &= \frac{(x-1)(x-1)}{(x-1)}\\ &= \frac{(x-1)(x-1)}{(x-1)}\\ &= x-1\end{aligned}$$

b. $$\begin{aligned}\frac{x^2-4}{x^2+4x+4} &= \frac{(x-2)(x+2)}{(x+2)(x+2)}\\ &= \frac{(x-2)(x+2)}{(x+2)(x+2)}\\ &= \frac{x-2}{x+2}\end{aligned}$$

c. $$\begin{aligned}\frac{x^2-5x+6}{x^2+2x-15} &= \frac{(x-3)(x-2)}{(x-3)(x+5)}\\ &= \frac{(x-3)(x-2)}{(x-3)(x+5)}\\ &= \frac{x-2}{x+5}\end{aligned}$$

d. $\dfrac{y^2-1}{y-1}=\dfrac{(y-1)(y+1)}{(y-1)}$

$=\dfrac{(\overset{1}{y-1})(y+1)}{(y-\underset{1}{1})}$

$=y+1$

2. a. $\dfrac{4}{xy}+\dfrac{5}{x}=\dfrac{4}{xy}+\left(\dfrac{5}{x}\times\dfrac{y}{y}\right)$

$=\dfrac{4}{xy}+\dfrac{5y}{xy}$

$=\dfrac{4+5y}{xy}$

b. $\dfrac{2}{x^2-1}+\dfrac{3}{x^2-2x+1}$

$=\dfrac{2}{(x-1)(x+1)}+\dfrac{3}{(x-1)(x-1)}$

$=\dfrac{2(x-1)}{(x-1)(x+1)(x-1)}+\dfrac{3(x+1)}{(x-1)(x+1)(x-1)}$

$=\dfrac{2(x-1)+3(x+1)}{(x-1)(x+1)(x-1)}$

$=\dfrac{2x-2+3x+3}{(x-1)(x+1)(x-1)}$

$=\dfrac{5x+1}{(x-1)(x+1)(x-1)}$

- The common denominator is $(x-1)(x+1)(x-1)$. To change the original denominators, the first fraction is multiplied by $\frac{x-1}{x-1}$ which is really 1, and the second fraction is multiplied by $\frac{x+1}{x+1}$ which is also 1.

- You cannot cancel in $\dfrac{2(x-1)+3(x+1)}{(x-1)(x+1)(x-1)}$.

c. $\dfrac{x^2-9}{x^2-5x+6}-\dfrac{3}{x-2}=\dfrac{(x-3)(x+3)}{(x-3)(x-2)}-\dfrac{3}{x-2}$

$=\dfrac{(\overset{1}{x-3})(x+3)}{(x-\underset{1}{3})(x-2)}-\dfrac{3}{x-2}$

$=\dfrac{x+3}{x-2}-\dfrac{3}{x-2}$

$=\dfrac{x+3-3}{x-2}$

$=\dfrac{x}{x-2}$

- Since $x-3$ is a common factor in the expression $\frac{(x-3)(x+3)}{(x-3)(x-2)}$, you can divide out the common factor $(x-3)$.

- Remember that $\dfrac{a}{b}-\dfrac{c}{b}=\dfrac{a-c}{b}$.

d. $\dfrac{2x}{4x^2+8x}-\dfrac{x}{2x+4}=\dfrac{2x}{2x(2x+4)}-\dfrac{x}{2x+4}$

$=\dfrac{\overset{1}{\cancel{2x}}}{\underset{1}{\cancel{2x}}(2x+4)}-\dfrac{x}{2x+4}$

$=\dfrac{1}{2x+4}-\dfrac{x}{2x+4}$

$=\dfrac{1-x}{2x+4}$

3. a. $99\times101=(100+1)(100-1)$

$=100^2-1^2$

$=10\,000-1$

$=9999$

b. $499\times501=(500-1)(500+1)$

$=500^2-1^2$

$=250\,000-1$

$=249\,999$

c. $506\times494=(500+6)(500-6)$

$=500^2-6^2$

$=250\,000-36$

$=249\,964$

4. a. $xy-5y+2x-10=xy+2x-5y-10$

$=(xy+2x)-(5y+10)$

$=x(y+2)-5(y+2)$

$=(x-5)(y+2)$

b. $3x^2-5xy+5y-3x=3x^2-3x-5xy+5y$

$=\left(3x^2-3x\right)-(5xy-5y)$

$=3x(x-1)-5y(x-1)$

$=(3x-5y)(x-1)$

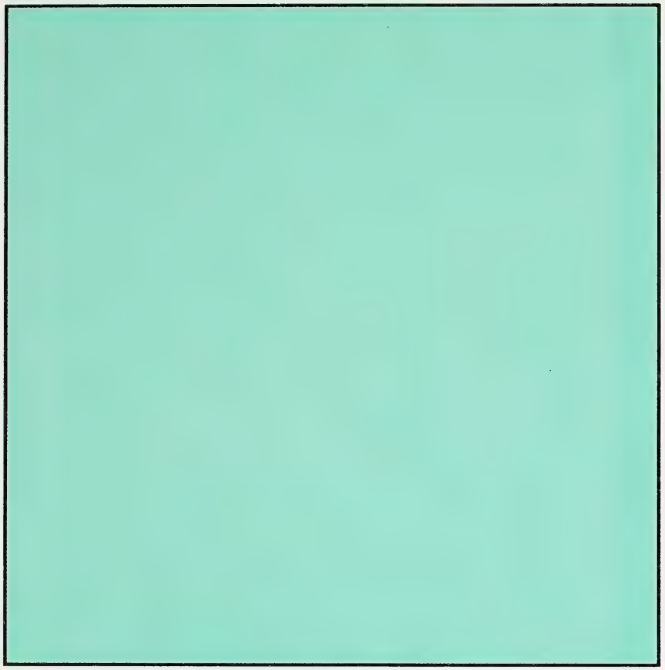
A difference of squares such as x^2-y^2 can be factored to give $(x+y)(x-y)$.

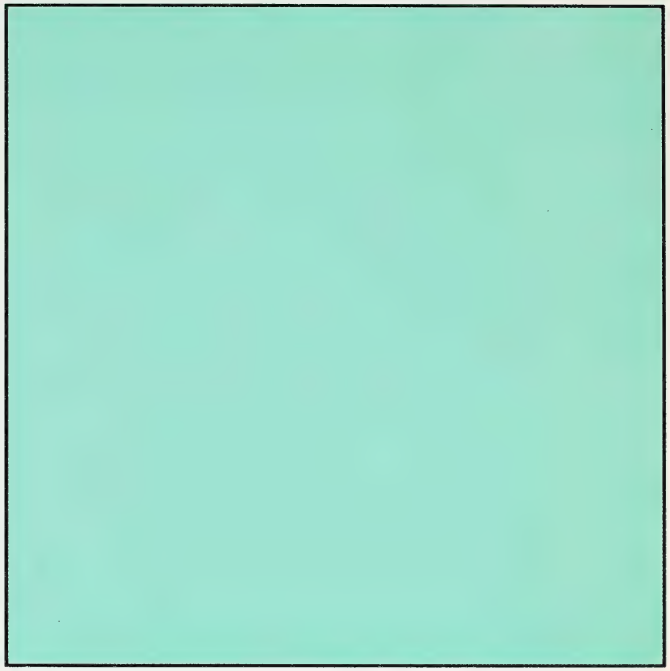
Appendix B
Tiles and Grids

Polynomial tiles: The following pages contain a set of polynomial tiles. The set includes six flats $\left(x^2\right)$, twenty strips (x), and fifty units (1). You are asked to cut out the tiles and use them to study polynomials in Topic 3. You should notice that the back of each tile has a diagonal line drawn across it. This means that this side represents a **negative** value.

Polynomial grids: Following the set of polynomial tiles is a set of polynomial grids. You can use these for extra practice or for any of the exercises that involve polynomial grids.

Polynomial Tiles

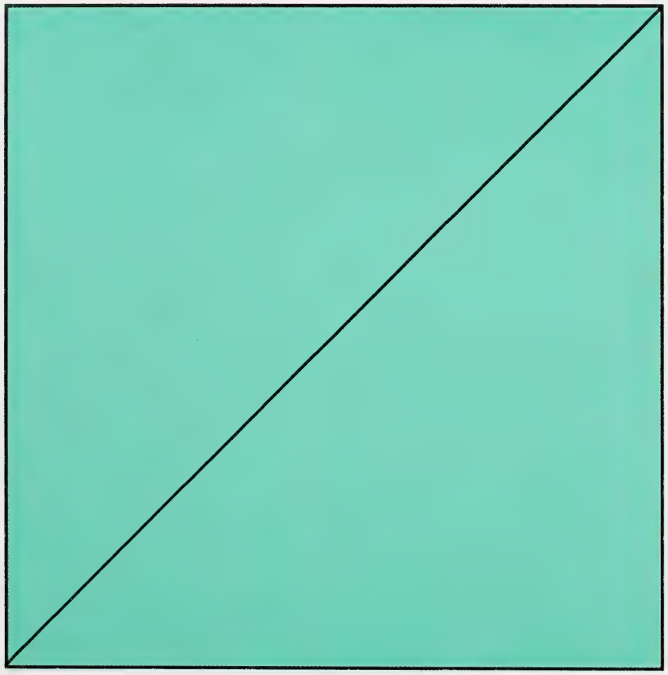


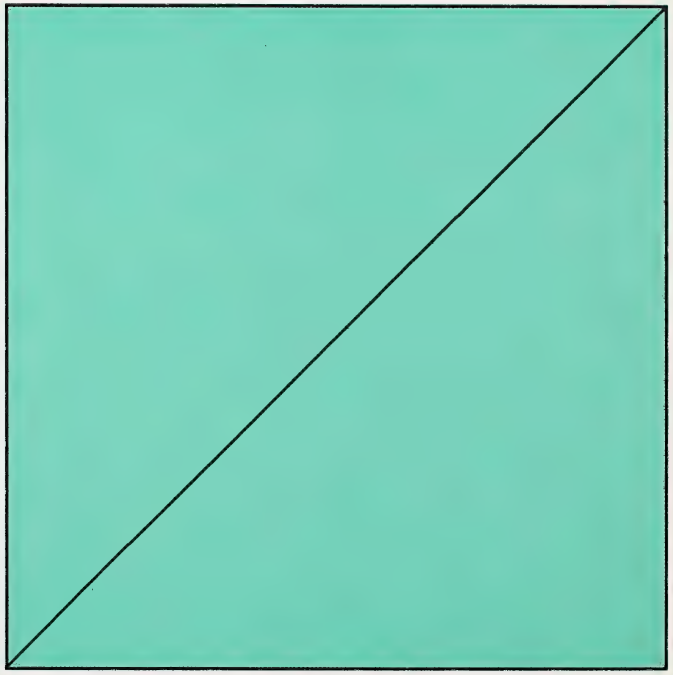


Each of these squares represents $+x^2$. Each one is called a flat.

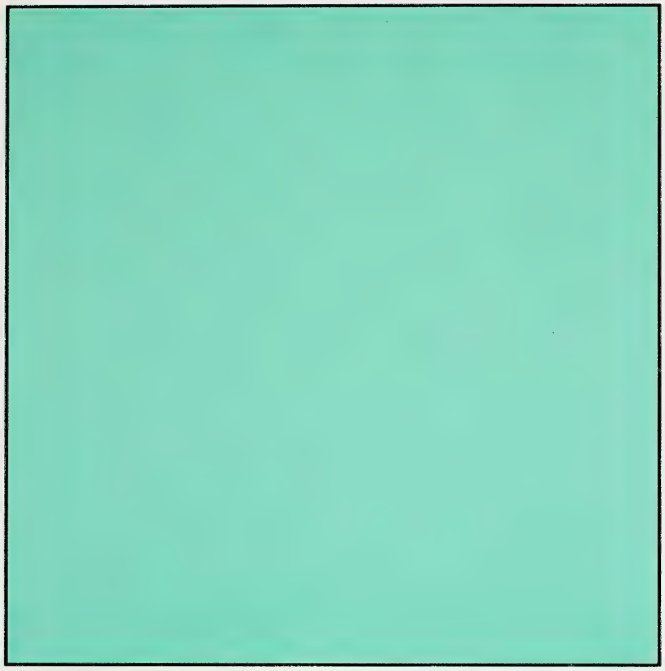
Polynomial Tiles

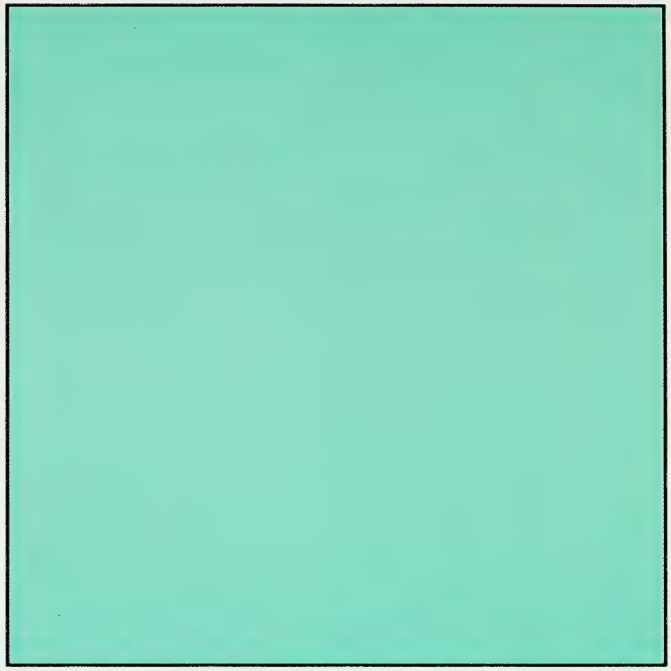
Each of these squares represents $-x^2$. Each one is called a negative flat.





Polynomial Tiles

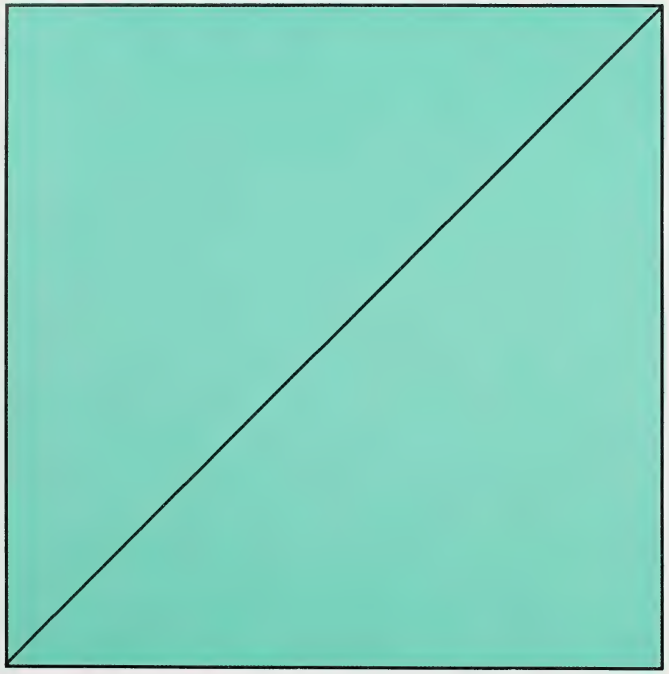


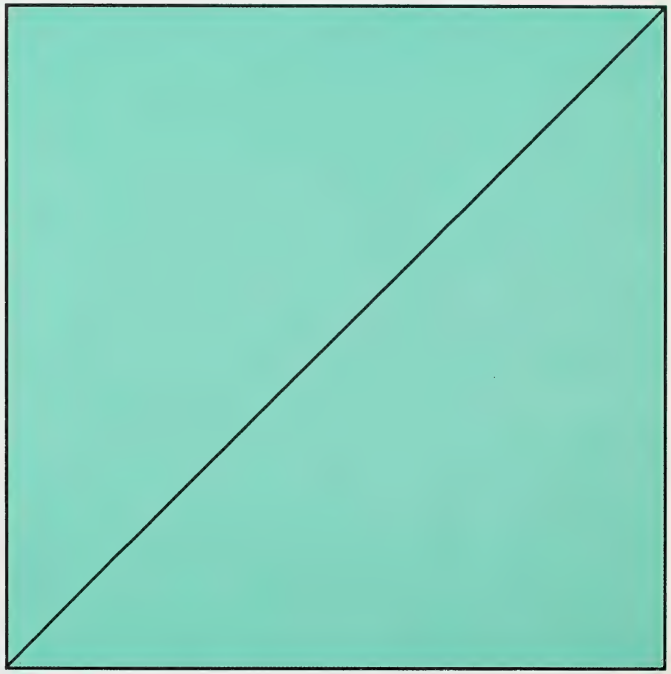


Each of these squares represents $+x^2$. Each one is called a flat.

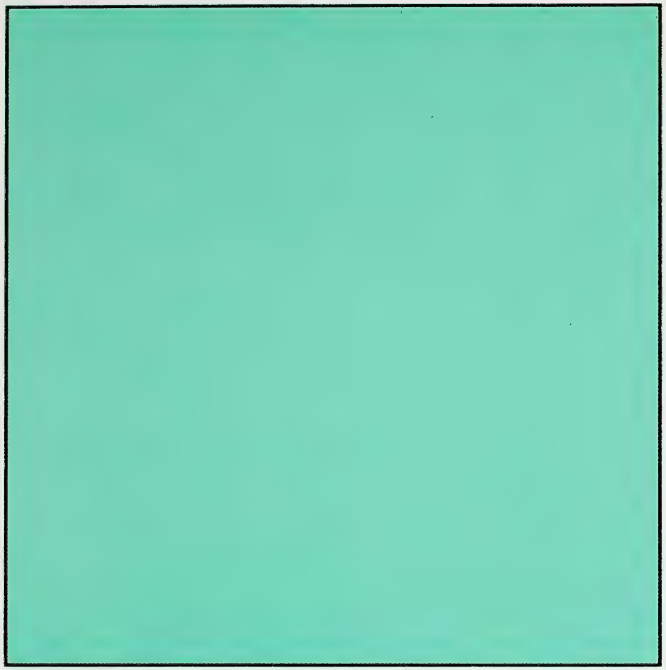
Polynomial Tiles

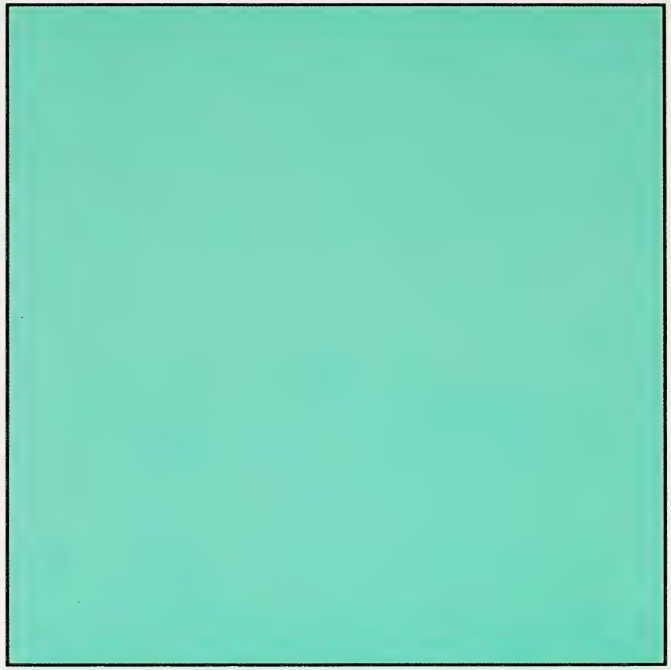
Each of these squares represents $-x^2$. Each one is called a negative flat.





Polynomial Tiles





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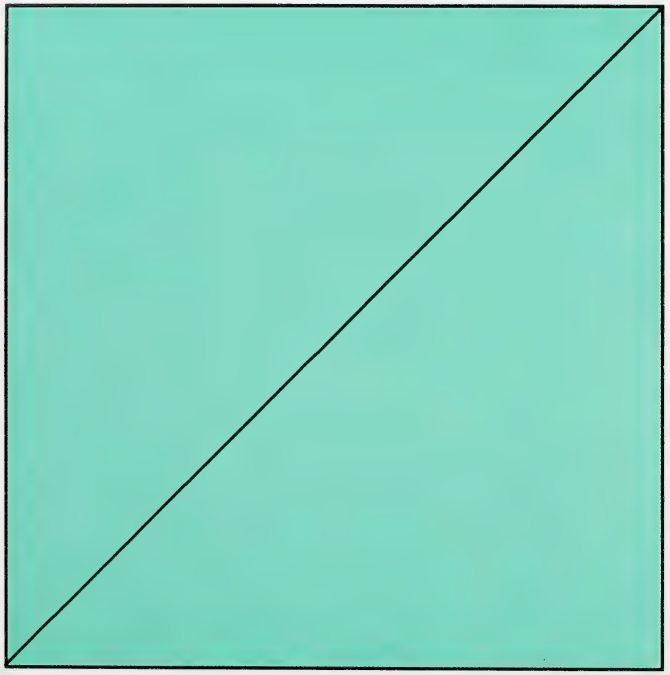
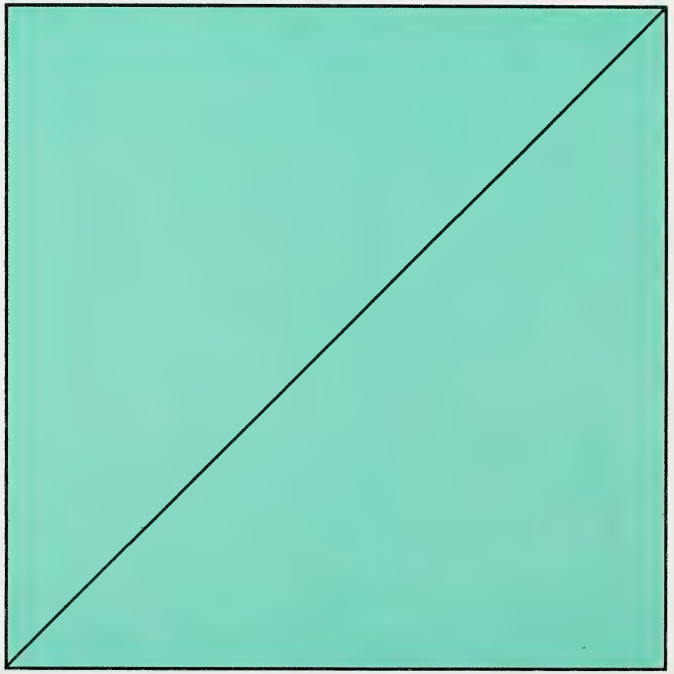
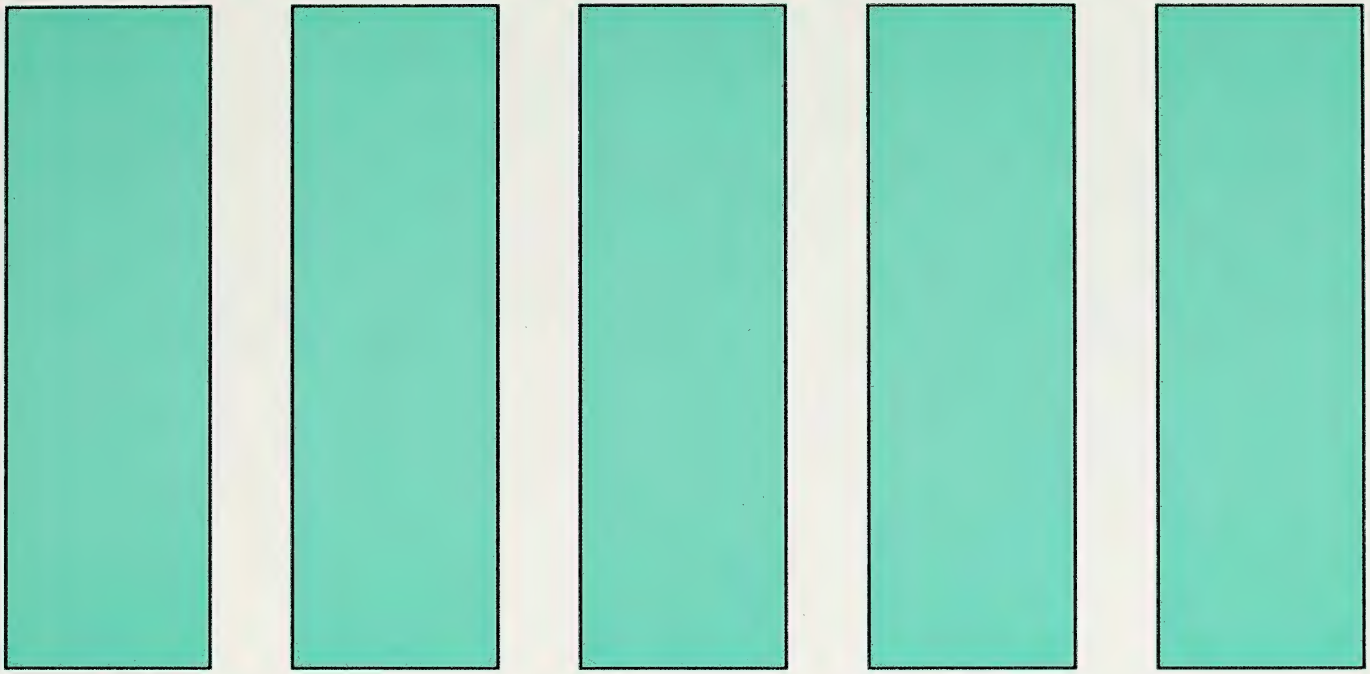
Polynomial Tiles

Each of these squares represents $-x^2$. Each one is called a negative flat.





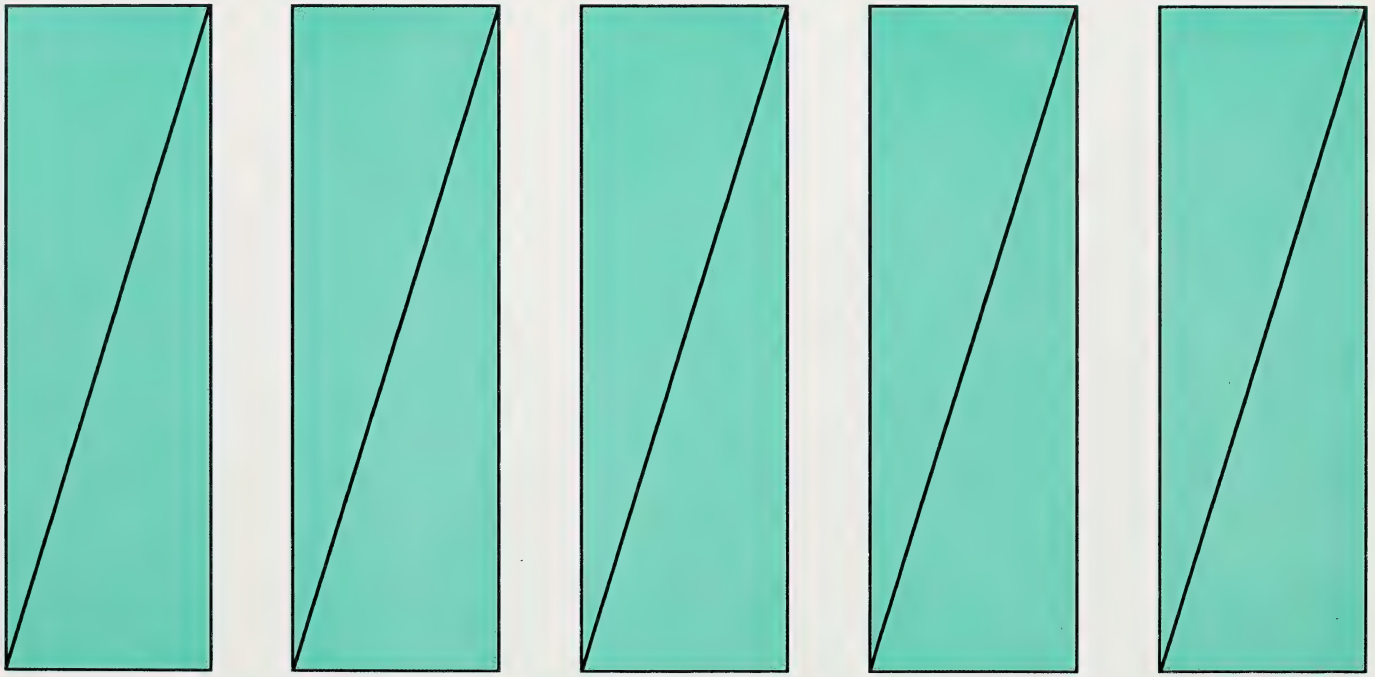
Polynomial Tiles



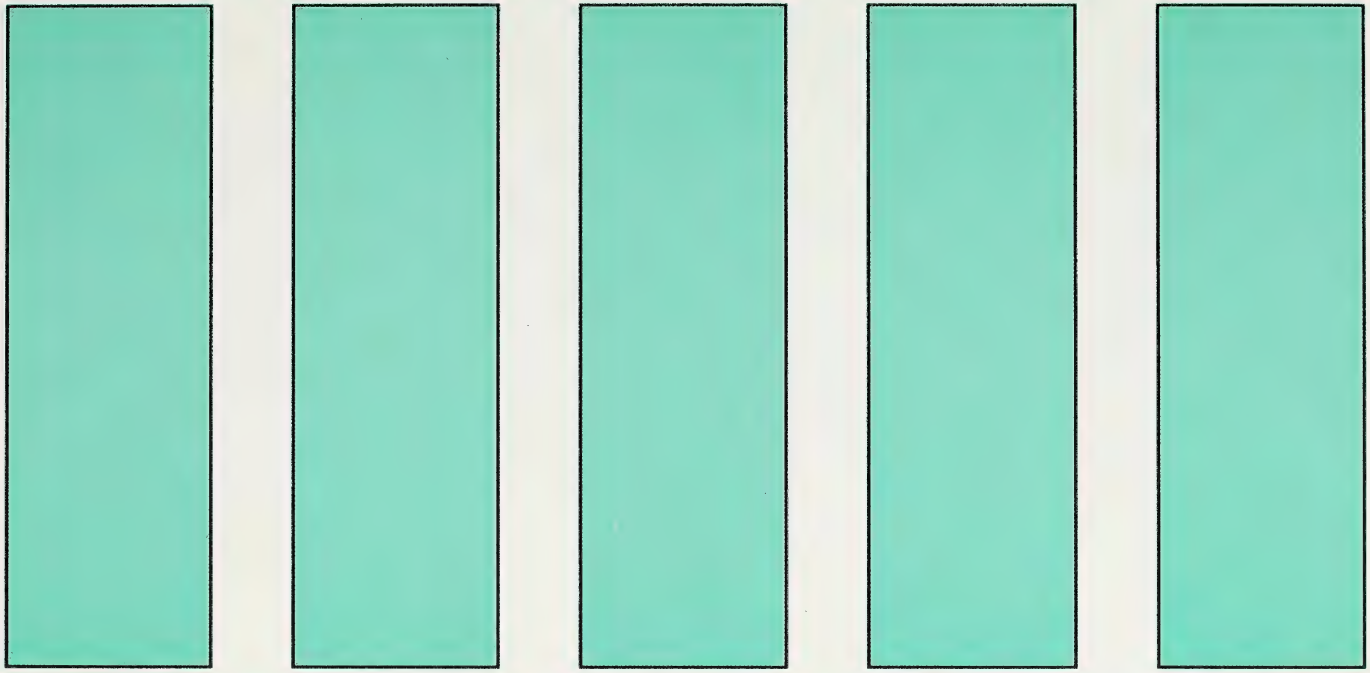
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



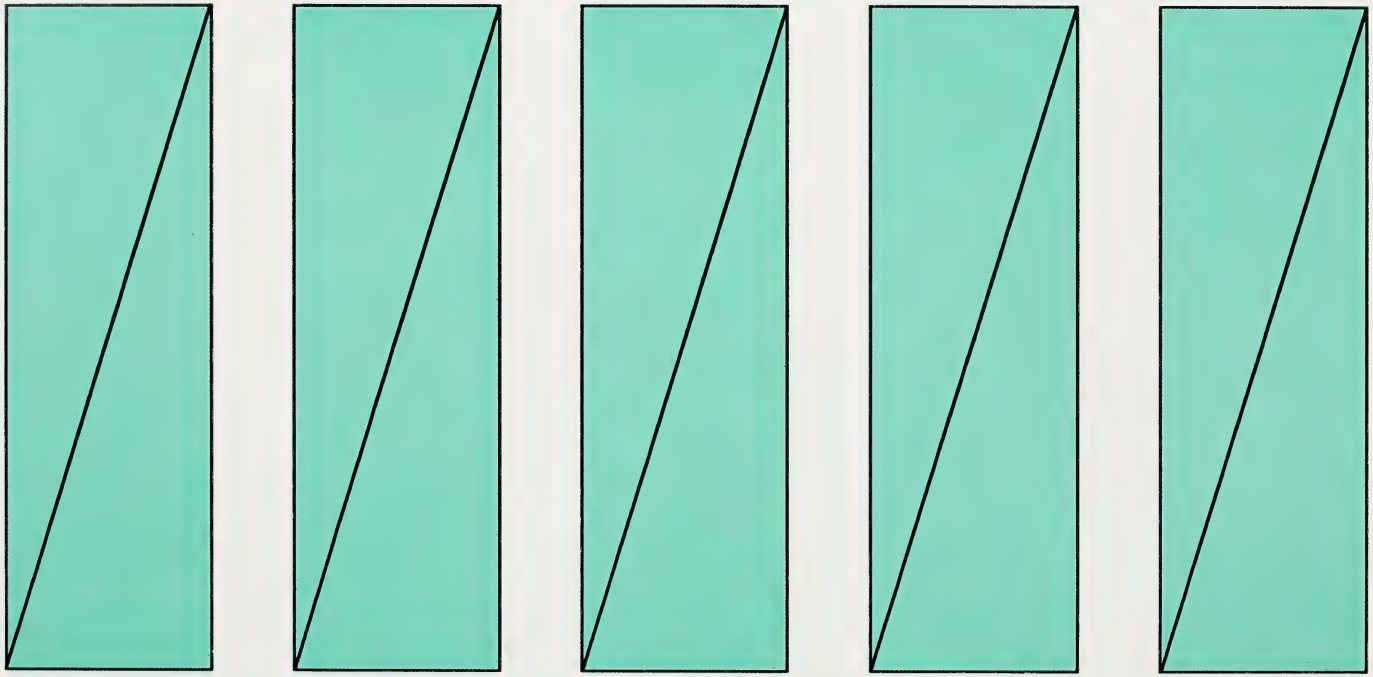
Polynomial Tiles



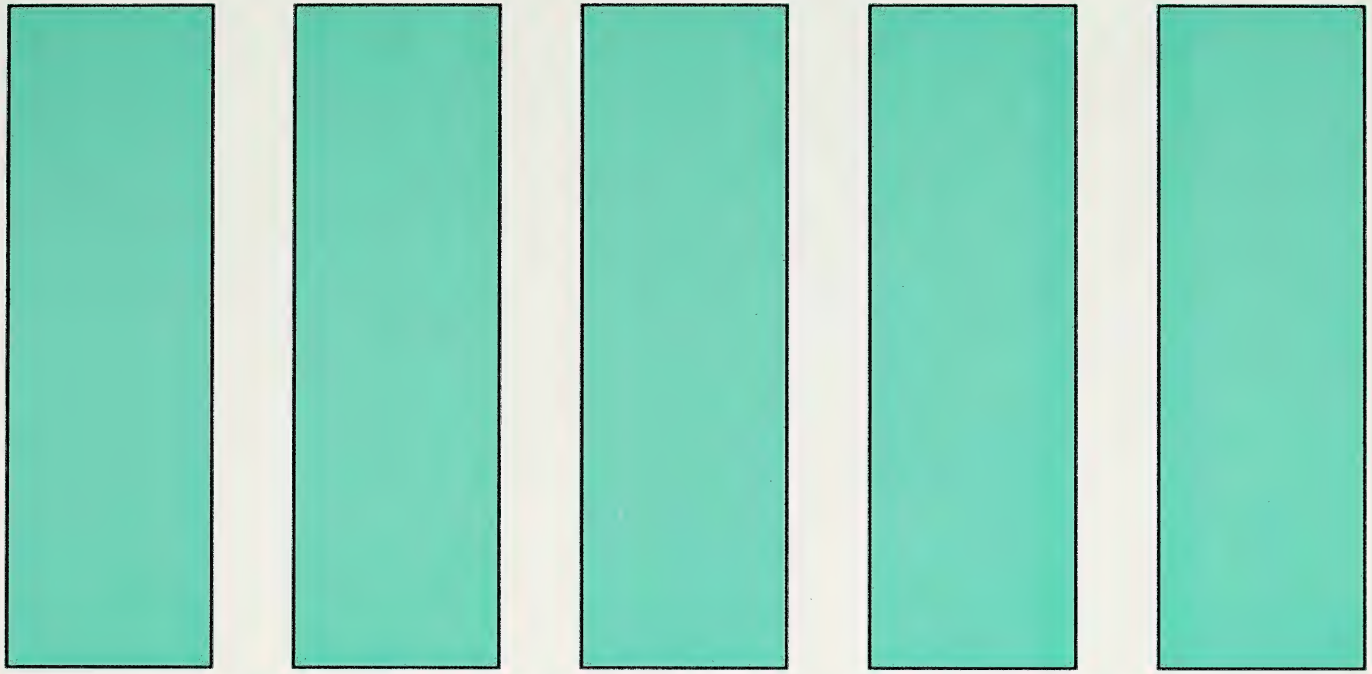
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



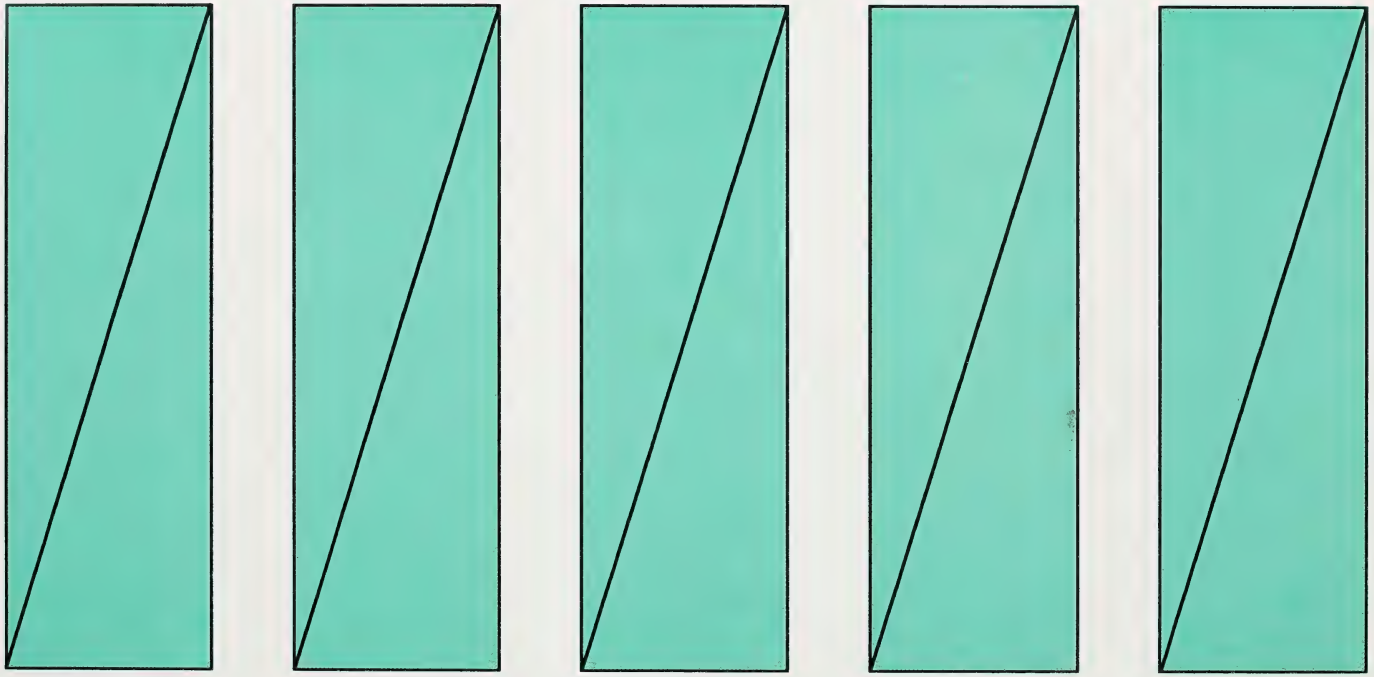
Polynomial Tiles



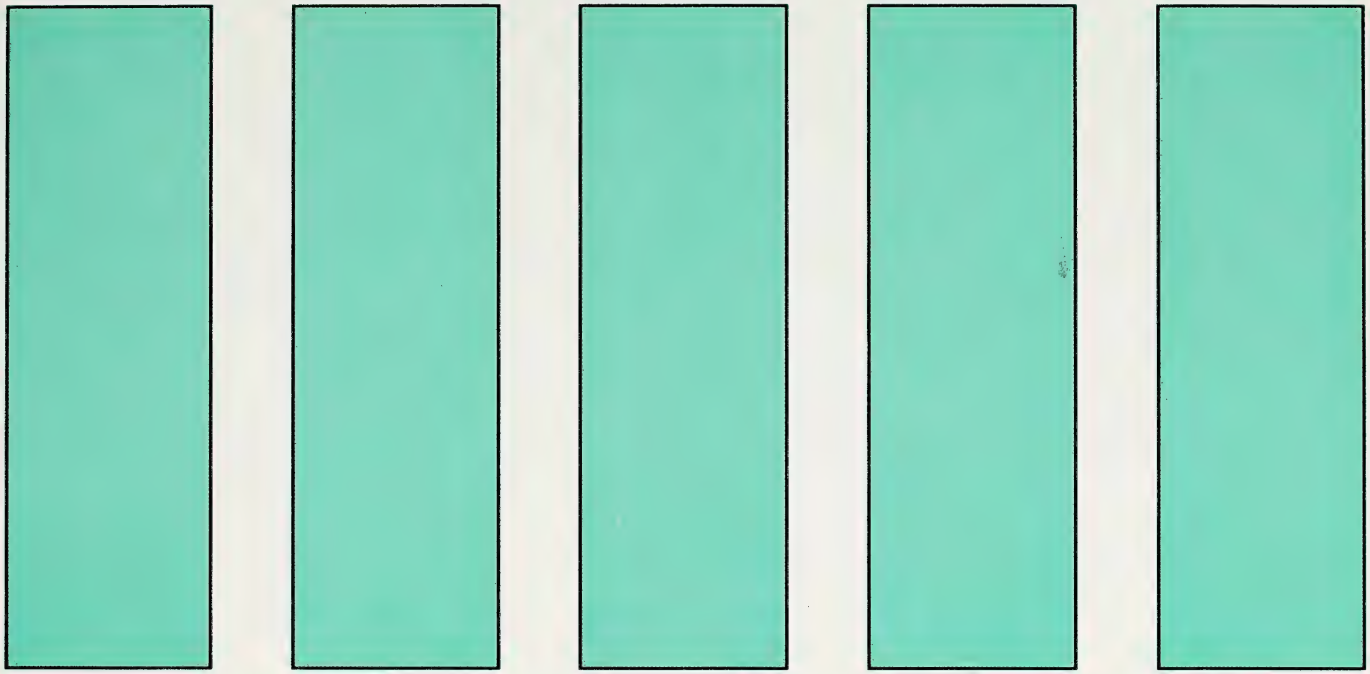
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



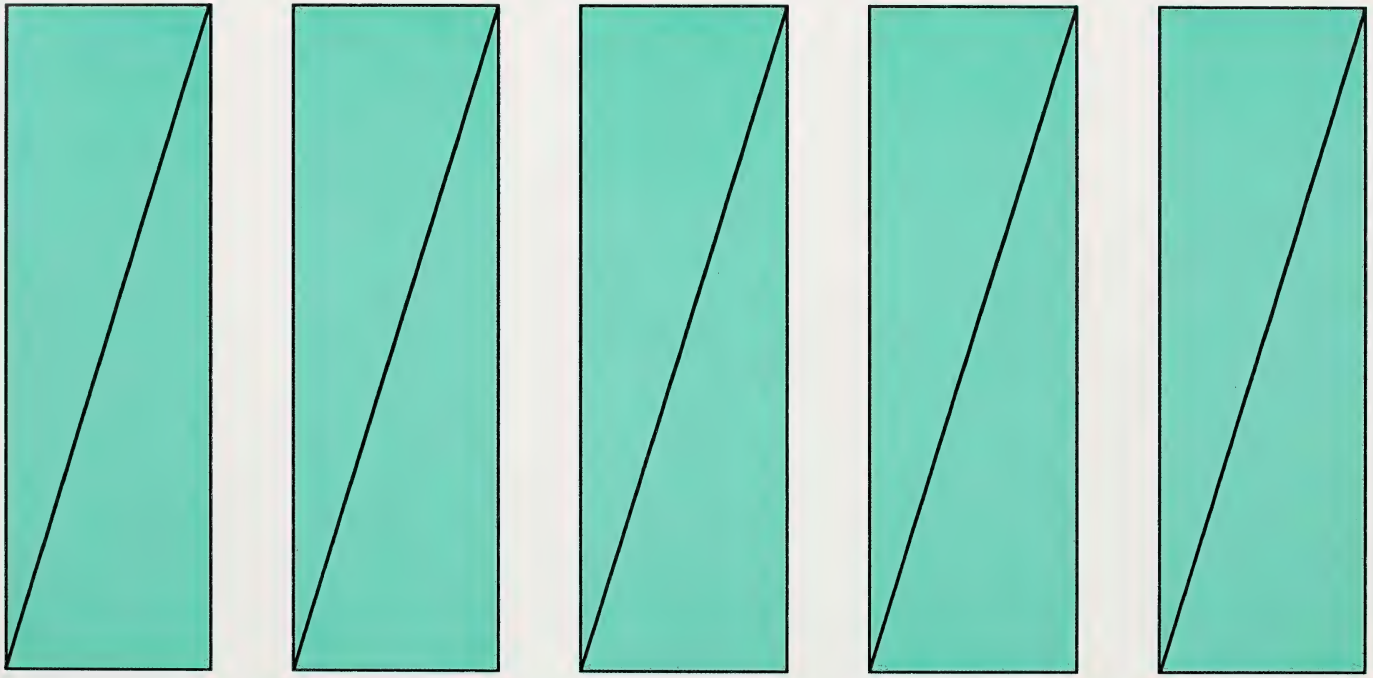
Polynomial Tiles



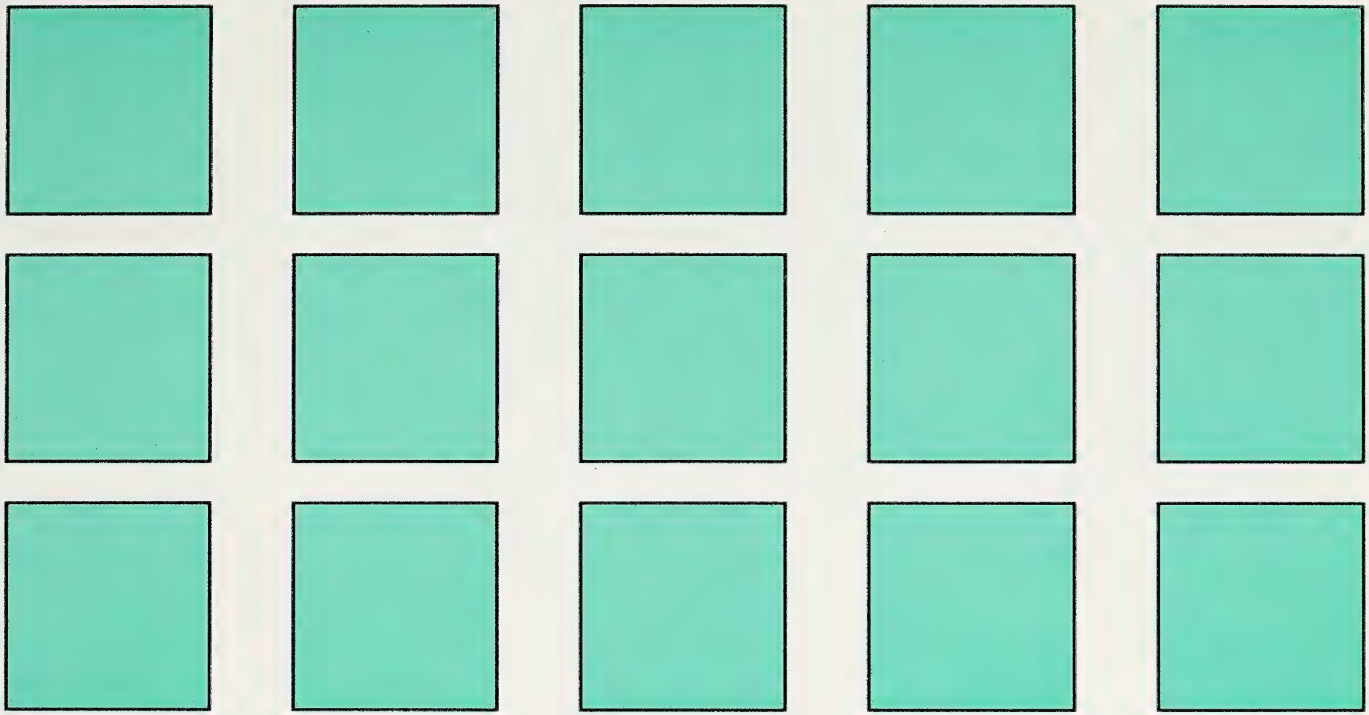
Each of these strips represents x. Each one is called a strip.

Polynomial Tiles

Each of these strips represents $-x$. Each one is called a negative strip.



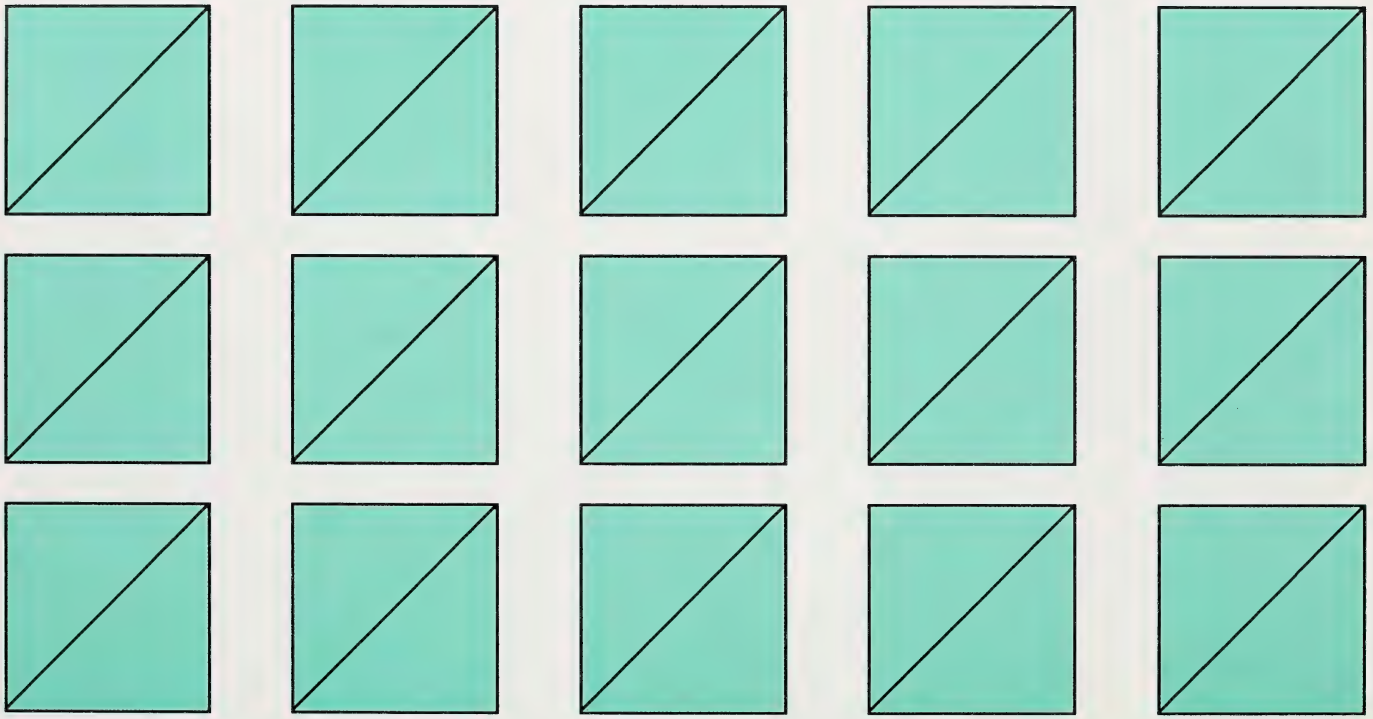
Polynomial Tiles



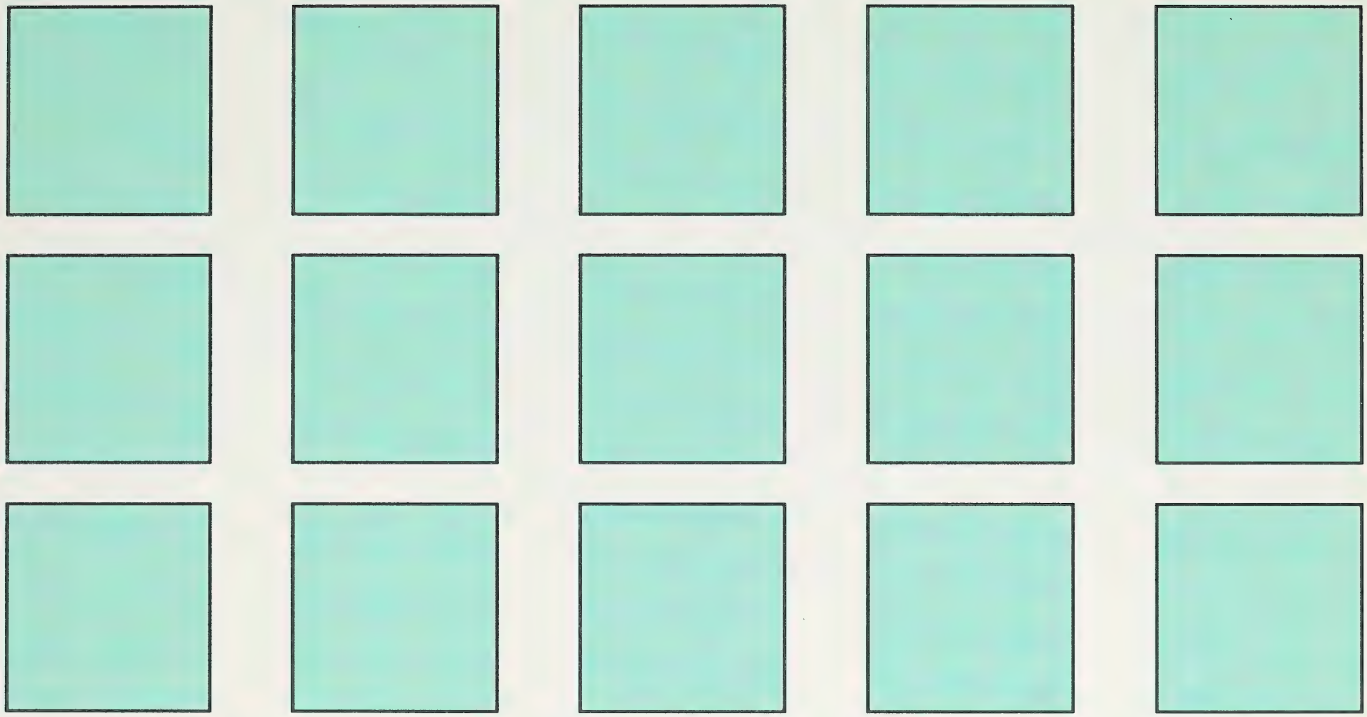
Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.



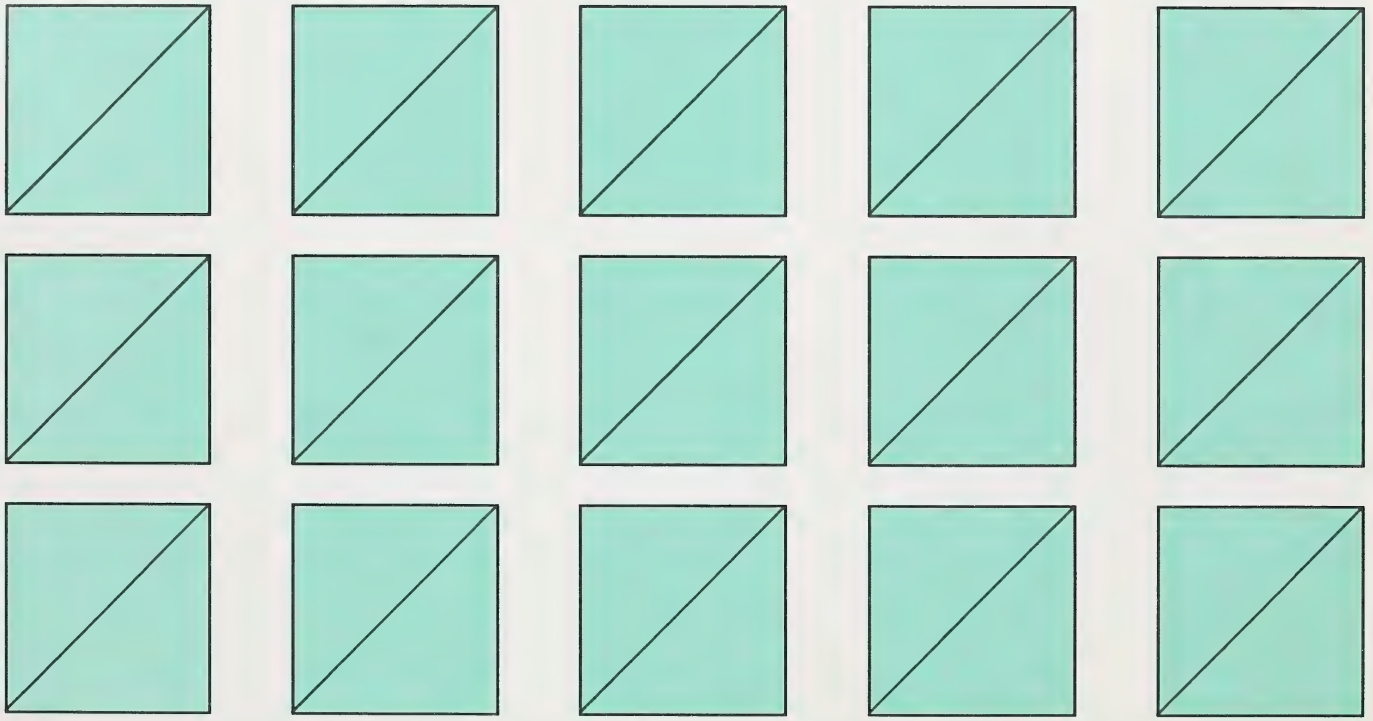
Polynomial Tiles



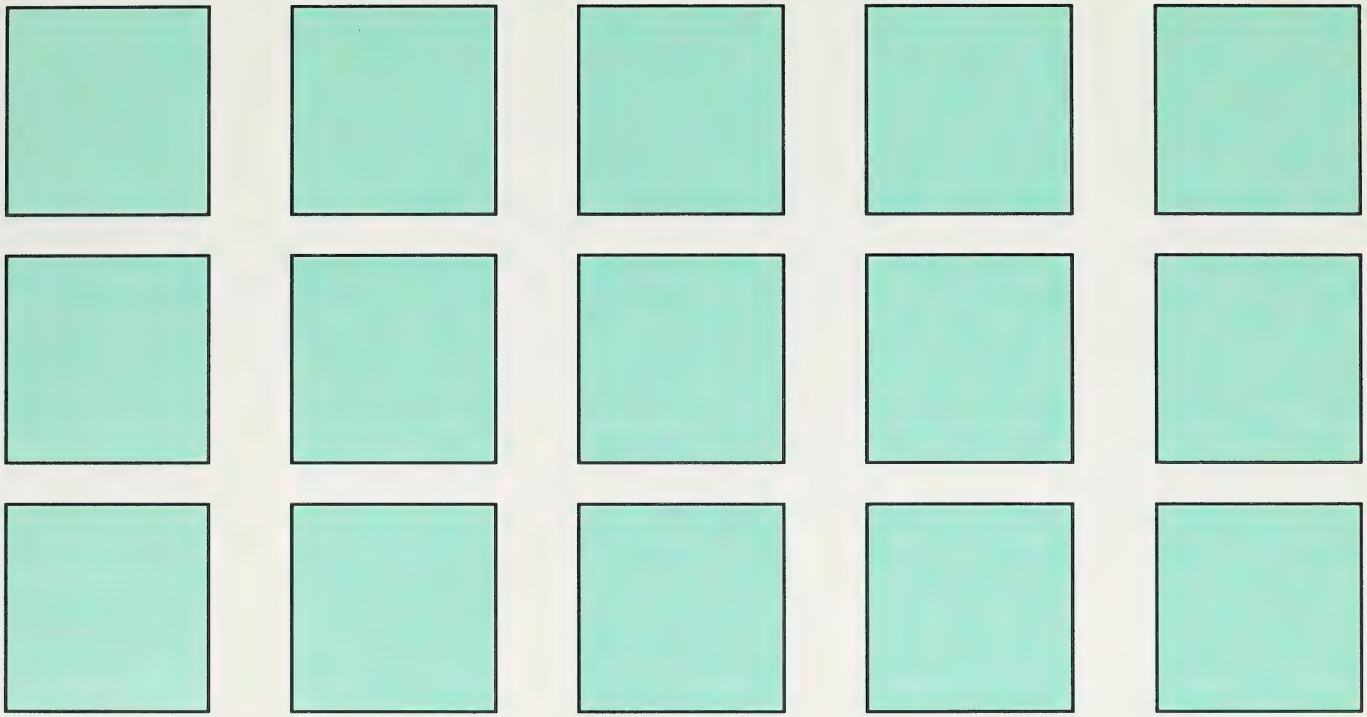
Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.



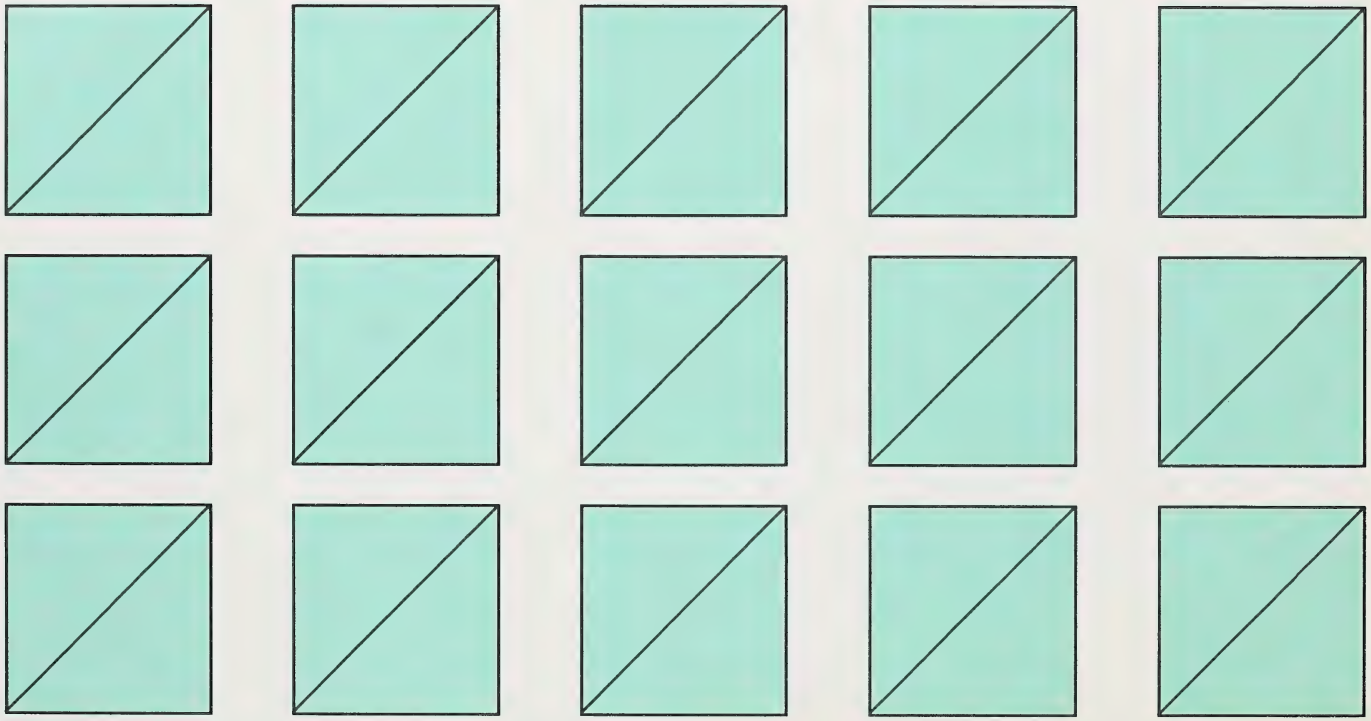
Polynomial Tiles



Each of these small squares represents 1. Each one is called a unit.

Polynomial Tiles

Each of these small squares represents -1. Each one is called a negative unit.

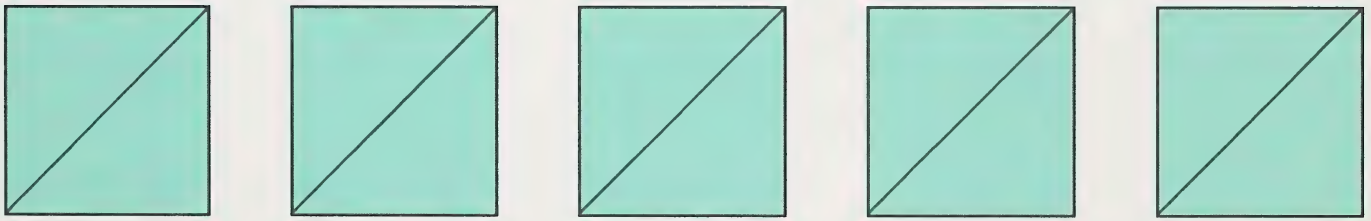


Polynomial Tiles

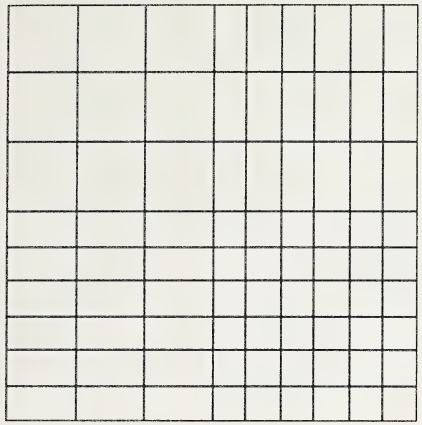
Each of these small squares represents 1. Each one is called a unit.

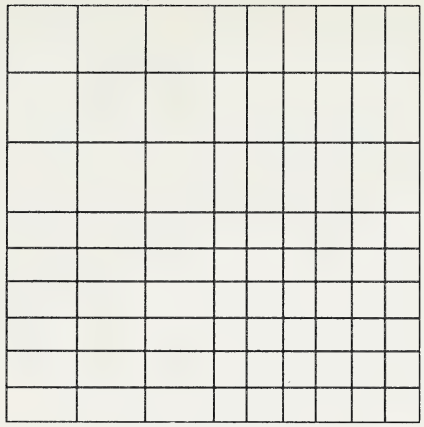
Polynomial Tiles

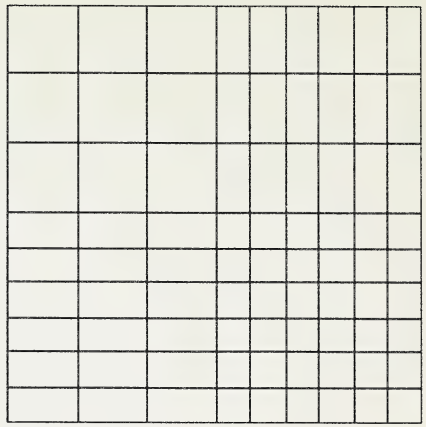
Each of these small squares represents -1. Each one is called a negative unit.

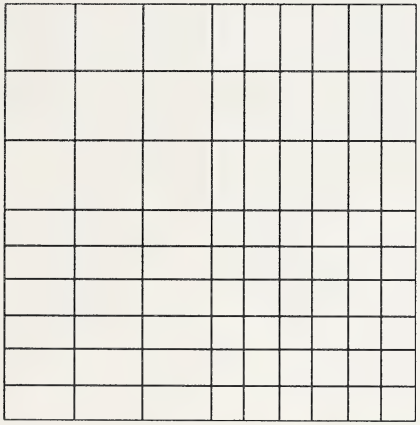


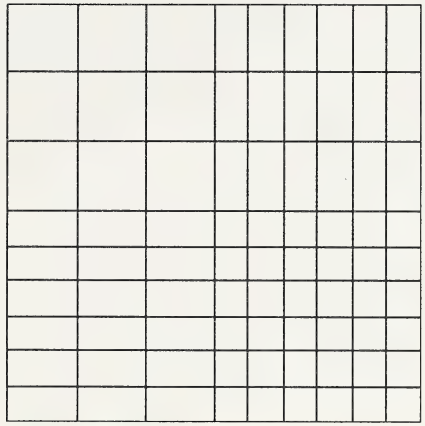
Polynomial Grids

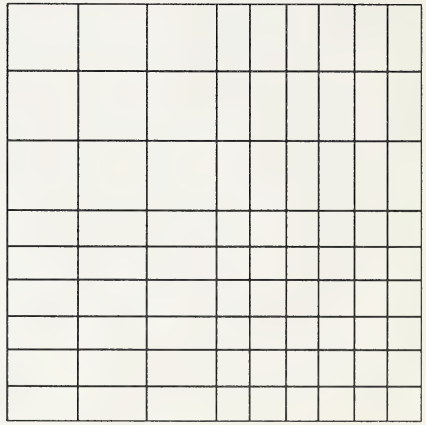




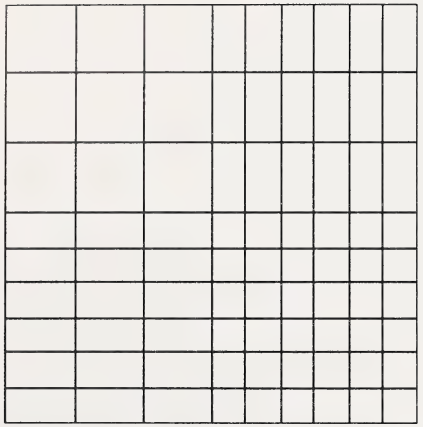


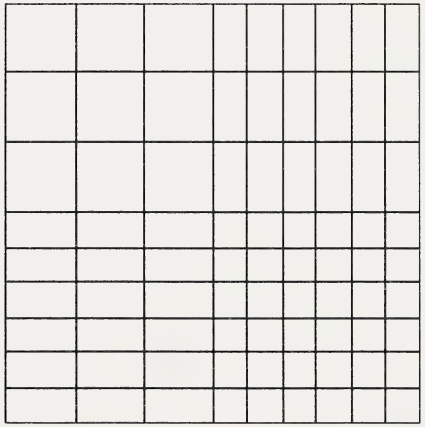


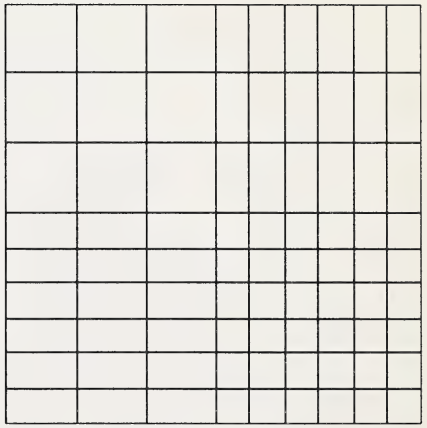


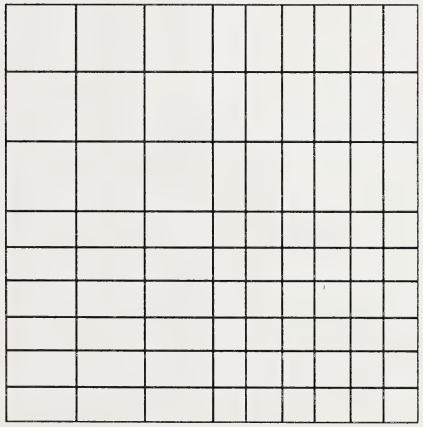


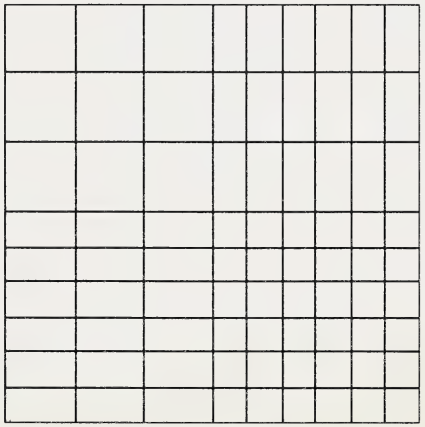
Polynomial Grids

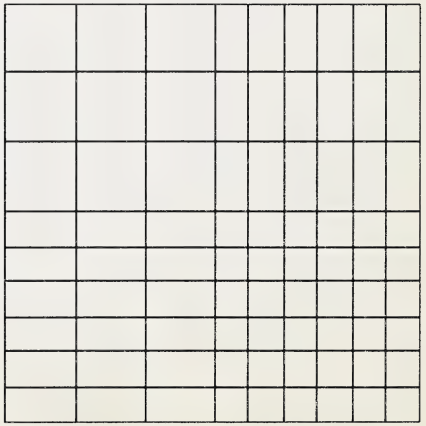




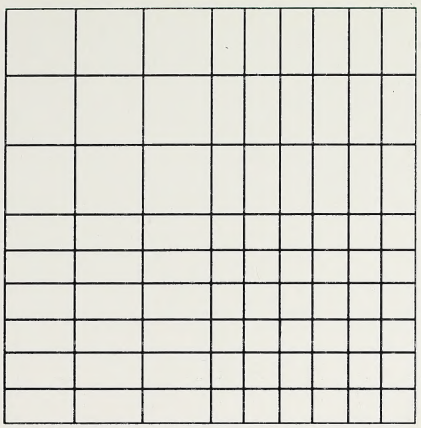


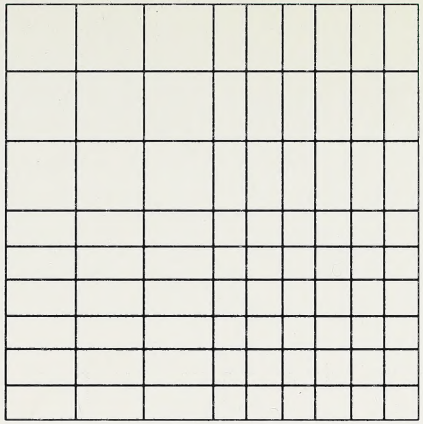


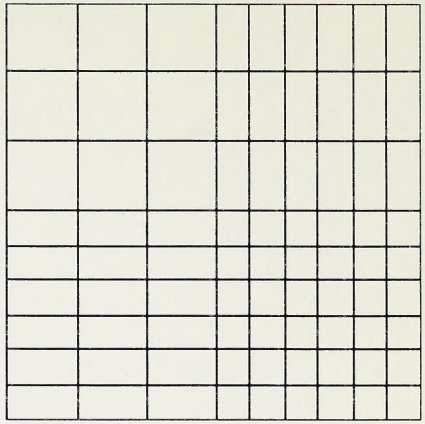


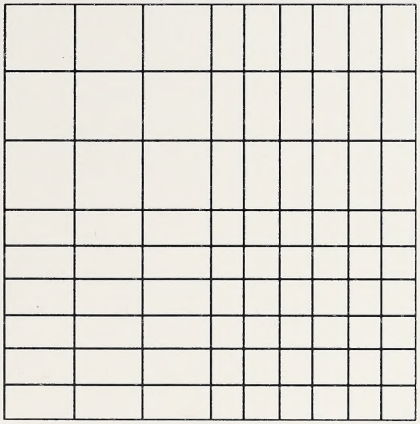


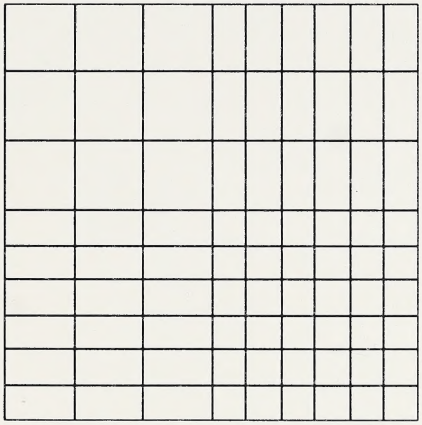
Polynomial Grids

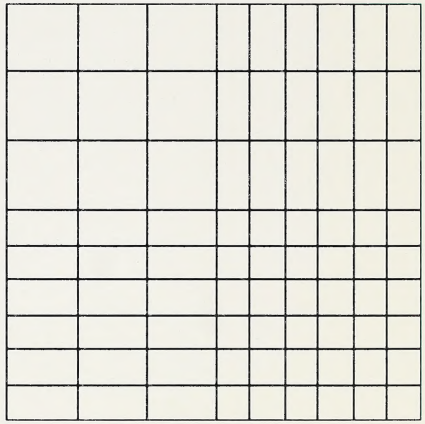












Mathematics 13

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